Math 121, Practice Questions on Chapter 1 and Complex Numbers

Except for questions 7, 8, 9, 13(b), 20(b) and 29, you should be able to do these problems without a calculator.

1. (a) Simplify \((3 - 2i)(4 + i)\) and write the complex number in standard form.
   (b) Simplify \(i^{223}\).

2. Write the complex number \(\frac{3 - 2i}{4 + 5i}\) in standard form.

3. (a) Solve the equation \(\frac{2x + 5}{3x - 1} = 1\).
   (b) Solve \(3|x - d| = c\) where \(c > 0\).

4. A worker can build a fence in 8 hours. With the help of an assistant, the fence can be built in 5 hours. How long would it take the assistant to build the fence alone?

5. A radiator contains 6 liters of 20% antifreeze solution. How much should be drained and replaced with pure antifreeze to produce a 50% antifreeze solution.

6. (a) Solve the equation \(A = \frac{1}{2} h (b_1 + b_2)\) for \(b_2\).
   (b) Solve the equation \(\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}\) for \(R_2\).
   (c) Solve the equation \(\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}\) for \(R\).

7. An investment of $2500 is made at an annual simple interest rate of 5.5%. How much additional money must be invested at an annual simple interest rate of 8% so that the total interest earned is 7% of the total investment?

8. How much pure gold should be melted with 15 grams of 14-karat gold to produce 18-karat gold? (Note: pure gold measures 24 karats, an alloy that is \(k\) karats is \(\frac{k}{24}\) · 100% gold. For example 12-karat gold is 50% gold.)

9. A messenger ran at 10 miles per hour from his residence to the King’s palace, he walked back at 4 miles per hour. The total trip took him \(8\frac{3}{4}\) hours. How far is it from the messenger’s residence to the King’s palace?

10. Solve the equation \(2x^2 + 3x = 4\) using the algebraic method of your choice.

11. Solve the quadratic equation \((x - 5)^2 - 9 = 0\) by factoring.
12. Solve the quadratic equation \( \frac{1}{2}x^2 + \frac{3}{4}x + 1 = 0 \) by using the quadratic formula.

13. (a) A gardener wishes to use 600 feet of fencing to enclose a rectangular region and subdivide the region into two smaller rectangles. See the figure below. The total enclosed area is 15,000 square feet. Find the dimensions of the enclosed area.

(b) The height of a ball thrown vertically in the air at 60 mph from a height of 7 feet is given by \( h(t) = -16t^2 + 88t + 7 \), where \( t \) is the number of seconds after it was released, and \( h(t) \) is the height in feet. Find how many seconds, if possible, it took the ball to reach a height of 130 feet. After how many seconds did the ball hit the ground?

14. (a) Solve the radical equation \( \sqrt{x + 1} - \sqrt{2x - 5} = 1 \). Check all proposed solutions.
(b) Solve the radical equation \( \sqrt{3x + 7} - 1 = x \). Check all proposed solutions.

15. Solve \( 4x^{\frac{3}{4}} = x^{\frac{1}{2}} \).

16. Solve the equation \( (3x - 5)^{\frac{2}{3}} + 6(3x - 5)^{\frac{1}{3}} = -8 \).

17. Pump 1 can drain a pool in 10 hours, while pump 2 can drain the same pool in 8 hours. How long would it take to drain the pool using both pumps?

18. Solve the equation \( x^3 - 6x^2 + 8x = 0 \) by factoring.

19. Solve the equation \( \sqrt{4x - 1} + 8 = 2x \). Check all proposed solutions.

20. (a) Solve the inequality \(-1 \leq -5x - 6 < 9\).

(b) Suppose the final test in a class is worth 30 percent of the overall grade. A student has an average of 92% going into the final. What percentage is needed on the final test for the student to finish with an overall average between 75% and 85%?
21. Solve the inequality $|4x - 2| \geq 10$.

22. Solve the inequality $|-x - 2| > -1$.

23. Solve the inequality $|5x + 1| < 0$

24. Solve the inequality $|6x + 3| < 15$.

25. Solve the inequality $x^2 < -x + 30$.

26. Solve the inequality $\frac{x + 1}{x - 2} \geq 2$ and write the answer in interval notation.

27. (A physics inequality) The equation $s = -16t^2 + v_0t + s_0$ gives the height $s$, in feet above ground level, at the time $t$ seconds, of an object thrown directly upward from a height $s_0$ feet above the ground and with an initial velocity of $v_0$ feet per second. A ball is thrown directly upward from a height of 33 feet above the ground with an initial velocity of 80 feet per second. Find the time interval during which the ball will be more than 97 feet above the ground.

28. Solve the inequality $\frac{(x - 2)(x + 1)}{x - 4} \geq 0$. Write your answer in interval notation.

29. The maximum load that a cylindrical column of circular cross section can support varies directly as the fourth power of diameter and inversely as the square of its height. If a column 2 feet in diameter and 10 feet high supports upto 6 tons, how much can a column 3 feet in diameter and 14 feet high support?

30. If the diameter of a column is doubled, by what factor does the amount of weight it can support increase? (refer to #29 for the maximum load relation)

31. If the height of a column is tripled and its diameter is halved, by what factor does the amount of weight it can support decrease? (refer to #29 for the maximum load relation)