Math 121, Practice for Test 4

The test will cover the following topics and sections from the text.

• Sections 9.1 and 9.2: systems of linear equations. Key types of problems included solving linear systems with 2 or 3 variables. Setting up and solving word problems involving linear systems of equations. Recognizing inconsistent, dependent and independent systems of equations.

• Sections 10.1, 10.2, and 10.3: Gaussian elimination, the algebra of matrices, inverses of matrices. Key types of problems include converting systems of equations to matrix form and solving the system using elementary row operations. Matrix addition and multiplication, writing linear systems in matrix form. Finding inverses of matrices 2 by 2 or 3 by 3. Recognizing singular matrices that don’t have inverses. Solving systems of linear equations using the matrix inverse.

• Sections 11.1 and 11.5: sequences and summation notation, the binomial theorem. Key types of problems include, using summation notation, finding terms of sequences, binomial expansions, and finding specific terms in a binomial expansion.

Some practice problems are as follows:

1. Solve each system of equations if possible.

   (a) \[
   \begin{cases}
   3x + 4y = -5 \\
   x - 5y = -8
   \end{cases}
   \]

   (b) \[
   \begin{cases}
   5x - 3y = 0 \\
   10x - 6y = 0
   \end{cases}
   \]

   (c) \[
   \begin{cases}
   3x + 6y = 11 \\
   2x + 4y = 9
   \end{cases}
   \]

2. A motorboat traveled a distance of 120 miles in 4 hours while traveling with the current. Against the current, the same trip took 6 hours. Find the rate of the boat in calm water and find the rate of the current.

3. A broker invests $25,000 of a clients’ money in two different municipal bonds. The annual rate of return on one bond is 6%, and the annual rate of return on the second bond is 6.5%. The investor receives a total annual interest payment of $1555 from the two bonds. Find the amount invested in each bond.

4. Solve the following systems of equations.

   (a) \[
   \begin{cases}
   3x + 4y - z = -7 \\
   x - 5y + 2z = 19 \\
   5x + y - 2z = 5
   \end{cases}
   \]

   (b) \[
   \begin{cases}
   2x + 3y - 6z = 4 \\
   3x - 2y - 9z = -7 \\
   2x + 5y - 6z = 8
   \end{cases}
   \]

   (c) \[
   \begin{cases}
   2x - 3y + 6z = 3 \\
   x + 2y - 4z = 5 \\
   3x + 4y - 8z = 7
   \end{cases}
   \]

5. The equation of a (nonvertical) plane can be written in the form \( z = ax + by + c \) where \( a, b, \) and \( c \) are numbers. Find the equation of the plane that contains the points \((2, 1, 1), (-1, 2, 12)\) and \((3, 2, 0)\).
6. A coin bank contains only nickels, dimes, and quarters. The value of the coins is $2. There are twice as many nickels as dimes and one more dime than quarters. Find the number of each coin in the bank.

7. Consider the following systems of equations. For each system, find the values of $k$ for which there (if possible), (i) one solution, (ii) no solution, (iii) infinitely many solutions?

(a) \[
\begin{align*}
3x + 4y - z &= -7 \\
5y + 2z &= 19 \\
kz &= 0
\end{align*}
\]

(b) \[
\begin{align*}
3x + 4y - z &= -7 \\
5y + 2z &= 19 \\
kz &= 10
\end{align*}
\]

8. Write the systems of equations in 1(a) and 4(a) as augmented matrices, and solve them using Gaussian elimination.

9. The following are augmented matrices for a systems of equations in the variables $x_1, x_2, x_3$ and $x_4$, find the solutions to the systems of equations.

(a) \[
\begin{bmatrix}
1 & 0 & 0 & 1 & 5 \\
0 & 1 & 0 & -3 & -2 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 0 & 0 & 1 & 5 \\
0 & 1 & 0 & -3 & -2 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 10
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 0 & 0 & 1 & 5 \\
0 & 1 & 0 & -3 & -2 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 5
\end{bmatrix}
\]

10. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 0 \\ 3 & 1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & -4 & 0 \\ -2 & 3 & 2 \end{bmatrix}$. Find the following, if possible.

(a) $AB$  
(b) $BA$  
(c) $A + B$  
(d) $5A - 3C$  
(e) $AC$.

11. Determine whether it is possible to find the product $AB$ for matrices of the given sizes, if so, determine the size of $AB$. Same question for $BA$.

(a) $A$ is a $3 \times 3$ matrix, $B$ is a $3 \times 1$ matrix.

(b) $A$ is a $3 \times 5$ matrix, $B$ is a $3 \times 5$ matrix.

(c) $A$ is a $5 \times 3$ matrix, $B$ is a $3 \times 5$ matrix.

(d) $A$ is a $3 \times 2$ matrix $B$ is a $2 \times 3$ matrix.

12. Write the following matrix equation as an equivalent system of equations.

\[
\begin{bmatrix}
1 & -3 & -2 \\
3 & 1 & 0 \\
2 & -4 & 5
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
6 \\
2 \\
1
\end{bmatrix}
\]
13. Find the inverses of the following matrices (if they exist).

(a) \[
\begin{bmatrix}
1 & 2 \\
-2 & -3
\end{bmatrix}
\]  
(b) \[
\begin{bmatrix}
1 & 2 & -1 \\
2 & 5 & 1 \\
3 & 6 & -2
\end{bmatrix}
\]  
(c) \[
\begin{bmatrix}
1 & 2 & -1 \\
3 & 6 & -2 \\
1 & 2 & 0
\end{bmatrix}
\]

14. Consider the following systems of equations:

(a) \[
\begin{align*}
x + y + 2z &= 4 \\
2x + 3y + 3z &= 5 \\
3x + 3y + 7z &= 14
\end{align*}
\]
(b) \[
\begin{align*}
x + y + 2z &= 4 \\
2x + 3y + 3z &= 8 \\
3x + 3y + 7z &= 13
\end{align*}
\]

Solve these systems using the fact that the inverse of \[
\begin{bmatrix}
1 & 1 & 2 \\
2 & 3 & 3 \\
3 & 3 & 7
\end{bmatrix}
\]  is \[
\begin{bmatrix}
12 & -1 & -3 \\
-5 & 1 & 1 \\
-3 & 0 & 1
\end{bmatrix}
\].

15. Find the first three terms and the 8th term of the sequence whose \(n\)th term is \(a_n = (-1)^n \cdot 2n - 1\).

16. Find the first three terms of the recursively defined sequence \(a_1 = 5\), and \(a_n = 2a_{n-1}\).

17. (a) Evaluate the sum \(\sum_{i=3}^{6} (-1)^i 2^i\).

(b) Write 8 + 10 + 12 + 14 + 16 + 18 in summation notation.

18. Expand the binomial \((x - y^3)^6\).

19. Find the fourth term of \((x + 2y)^{12}\).

20. Find the term that contains \(b^9\) in the expansion of \((a - b^3)^8\).