Math 121, Test 2, October 25, 2006

Instructor: Name: Hint and Answers

Instructions. Do each of the following 12 problems. Each problem is worth 5pts. Show all appropriate details in your solutions. No calculators are not allowed on the first 6 problems.

1. (a) Find the midpoint of (1, −3) and (5, −7).

\[ MP = \left( \frac{1+5}{2}, \frac{-3-7}{2} \right) = (3, -5) \]

(b) Find the distance between (1, 0) and (4, −5). (Leave answer in radical form)

\[ d = \sqrt{(4-1)^2 + (-5-0)^2} = \sqrt{3^2 + 5^2} = \sqrt{9+25} = \sqrt{34} \]

2. Find the center and radius of the circle \( x^2 + y^2 - 10x + 2y + 25 = 0 \).

\[ x^2 - 10x + 25 + y^2 + 2y + 1 = 0 + 1 \quad (\text{complete squares}) \]

\[ (x-5)^2 + (y+1)^2 = 1 \]

\[ \text{Center is} (5,-1) \]

\[ \text{Radius is} 1 \]

3. Consider the piecewise defined function \( f(x) = \begin{cases} \frac{x+2}{3-x} & \text{if } x < 3; \\ 100 & \text{if } 3 \leq x \leq 5; \\ 3x - 1 & \text{if } x > 5. \end{cases} \)

Find: (a) \( f(1) = \frac{1+2}{3-1} = \frac{3}{2} \)

(b) \( f(3) = 100 \)

(c) \( f(7) = \frac{3(7)-1}{3(7)-1} = \frac{20}{20} \)

(d) \( f(t+1) \) if \( t > 4 \)

\[ f(t+1) = \frac{3(t+1)-1}{3t+2} \]
4. Find the equation of the line through the points \((3, -5)\) and \((1, -9)\). Write your answer in slope-intercept form.

\[
m = \frac{-9 - (-5)}{1 - 3} = \frac{-4}{-2} = 2
\]

\[
\therefore y + 5 = 2(x - 3)
\]

\[
[ y = 2x - 11 ]
\]

5. Consider the quadratic function \(f(x) = 3x^2 - 12x + 5\).

(a) Does the graph of \(y = f(x)\) open upward or downward?

\[3 > 0 \quad \therefore \text{upward}\]

(b) Does \(f\) have a maximum or does \(f\) have a minimum?

\(f\) has a \underline{minimum}\n
(c) Find the vertex of the graph \(y = 3x^2 - 12x + 5\).

\[h = \frac{-(12)}{2(3)} = \frac{-12}{6} = 2 \quad k = 3(2^2) - 12(2) + 5 = 12 - 24 + 5 = -7\]

\[\text{vertex } \bar{h} : (2, -7)\]

(d) Find the range of the function \(f(x)\). Write your answer in interval notation.

\[\text{range: } [-7, \infty)\]

6. For each of the following equations, determine if its graph is symmetric with respect to the \(x\)-axis, \(y\)-axis or origin. Note that a graph may have no symmetry or more than one type of symmetry.

(a) \(x^2 + y^2 = 25\) \quad \text{circle center origin, } r = 5

\[\text{symmetric with respect to } x\text{-axis}, y\text{-axis and origin}\]

(b) \(y = |x| - x\)

\[
\begin{cases}
-y = 1 - x - x \quad \text{diff eq} \\
y = 1 - x - (-x) \quad \text{diff eq} \\
-y = 1 - x - (-x) \quad \text{diff eq}
\end{cases}
\]

\[\text{none of these symmetric}\]

(c) \(x + 24 = y^2\).

\[
(-y)^2 = x + 24 \quad \Rightarrow \quad y^2 = x + 24 \quad \text{Same eq}
\]

\[\text{symmetric w.r.t. } x\text{-axis}.
\]

\[-x + 24 = y^2 \quad (\text{different eq } \Rightarrow \text{not symmetric to } y\text{-axis})
\]

\[-x + 24 = (-y)^2 \Rightarrow -x + 24 \neq y^2 \quad (\text{different eq } \Rightarrow \text{not symmetric to origin})\]
7. Use the graph of \( g \) to sketch (a) \( y = g(x - 2) + 3 \) and (b) \( y = 3 - g(x) \).

(a) **Shift: 2 units right, 3 units up.**

(b) **Reflect over y-axis. Shift 3 units up.**

8. Let \( f(x) = \sqrt{x + 5} \) and \( g(x) = \sqrt{3 - x} \).

(a) Find the domain of \( \frac{f}{g} \); write your answer in interval notation.

\[
\frac{f(x)}{g(x)} = \frac{\sqrt{x + 5}}{\sqrt{3 - x}}
\]

\[ x \geq -5 \quad \text{and} \quad x < 3 \quad \text{but} \quad x \neq 3 \quad \Rightarrow \quad \boxed{[-5, 3)} \]

(b) Find \((f + g)(-1), (f - g)(-1), (fg)(-1)\) and \((f \circ g)(-1)\).

\[
(f(-1)) = 2, \quad (g(-1)) = 2
\]

\[
(f + g)(-1) = 4, \quad (f - g)(-1) = 0, \quad (fg)(-1) = 2 \cdot 2 = 4
\]

\[
(f \circ g)(-1) = f(g(-1)) = \sqrt{2 + 5} = \sqrt{7}
\]

9. Find the difference quotient \( \frac{f(x + h) - f(x)}{h} \) for \( f(x) = x^2 - 3x + 5 \).

\[
\frac{f(x + h) - f(x)}{h} = \frac{(x + h)^2 - 3(x + h) + 5 - (x^2 - 3x + 5)}{h}
\]

\[
= \frac{x^2 + 2xh + h^2 - 3x - 3h + 5 - x^2 + 3x - 5}{h}
\]

\[
= \frac{2xh + h^2 - 3h}{h}
\]

\[
= \frac{2x + h - 3}{1}
\]
10. A rectangle has a length of \( l \) feet and a perimeter of 50 feet. 
(a) Write the width \( w \) of the rectangle as a function of its length. 
\[
2l + 2w = 50 \quad \therefore \quad w = \frac{50 - 2l}{2} = 25 - l
\]
(b) Write the area \( A \) of the rectangle as a function of its length. 
\[
A = lw = l\left(\frac{50 - 2l}{2}\right) = l(25 - l) = -l^2 + 25l
\]

11. A magazine company had a profit of $320,000 when it had 50,000 subscribers. When it obtained 55,000 subscribers, it had a profit of $355,000. Assume the profit \( P \) is a linear function of the number of subscribers \( s \).
(a) Find the function \( P \). 
\[
m = \frac{355000 - 320000}{55000 - 50000} = \frac{35000}{5000} = 7
\]
\[
\therefore \quad 320000 = 7(50000) + b \quad \Rightarrow \quad b = -30000
\]
\[
\therefore \quad P(s) = 7s - 30000
\]
(b) What will the profit be when the company obtains 63,000 subscribers? 
\[
P(63000) = 7(63000) - 30000 = 411,000
\]
(c) What is the number of subscribers needed to break even? 
\[
7s - 30000 = 0 \quad \Rightarrow \quad s = \frac{30000}{7} \approx 4285.7
\]
\[
\therefore \quad \text{needs 4286 subscribers to break even}
\]

12. The height of a ball thrown off of a cliff is given in feet by \( h(t) = -16t^2 + 104t + 250 \) where \( t \geq 0 \) is measured in seconds. 
(a) Find the time when the ball reaches its maximum height. 
\[
t = -\frac{b}{2a} = -\frac{104}{2(-16)} = 3.25 \text{ seconds (After 3.25 seconds)}
\]
(This is a quadratic equation, maximum occurs at vertex)
(b) Find the maximum height attained by the ball. 
\[
h\left(\frac{13}{4}\right) = -16\left(\frac{13}{4}\right)^2 + 104\left(\frac{13}{4}\right) + 250 = -16\left(\frac{13}{4}\right) + 2(169) + 250 = 169 + 250
\]
\[
= 419.7
\]
\[
\text{Ball reached a maximum height of 419.7 feet after 3.25 seconds}
\]