Name: ________________________________

Instructions. Do each of the following questions. The first five question are worth 2 points each, and the last eight questions are worth 5 points each. You must show all appropriate work to obtain full credit. Good Luck!

1. Find the system of equations that is needed to find $a$, $b$ and $c$ in the equation of the parabola $y = ax^2 + bx + c$ whose graph passes through the points $(2, 4)$, $(0, -4)$ and $(-2, 4)$. Do not solve the system of equations you found.

2. The augmented matrix
\[
\begin{bmatrix}
1 & 0 & 1 & -3 \\
0 & 1 & -1 & 0 \\
0 & 0 & k & -2
\end{bmatrix}
\]
represents a system of equations in the variables $x$, $y$, $z$. For which value(s) of $k$ is there no solution?

3. The augmented matrix
\[
\begin{bmatrix}
1 & 0 & 1 & -3 \\
0 & 1 & -1 & 0 \\
0 & 0 & k & -2
\end{bmatrix}
\]
represents a system of equations in the variables $x$, $y$, $z$. For which value(s) of $k$ is there exactly one solution?

4. Find the 17th term of the sequence whose $n$th term is given by $a_n = (-1)^n + 3$.

5. Let $A = \begin{bmatrix} 12 & -1 & 0 \\ -5 & 1 & 1 \\ 2 & -1 & 2 \\ -2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 1 & -1 & 0 \\ 4 & 0 & 2 & -2 & 0 \\ -2 & 4 & -1 & 2 & -2 \end{bmatrix}$, and let $C = AB$. Find $c_{43}$ (this is the only entry of $C$ you need to find, do not find the others).
6. A canoeist can row 12 miles with the current in 2 hours. Rowing against the current, it takes the canoeist 4 hours to travel the same distance. Find the rate of the canoeist in calm water and the rate of the current.

7. Find the sum \( \sum_{k=2}^{4} \frac{(-1)^k k}{3^{k-1}} \). Write your answer as a simplified fraction.

8. Let \( A = \begin{bmatrix} 0 & -3 \\ 1 & -3 \end{bmatrix} \), \( B = \begin{bmatrix} -2 & -1 & 4 \\ 3 & 1 & 0 \end{bmatrix} \), and \( C = \begin{bmatrix} 1 & -1 & 4 \\ 0 & -5 & -3 \end{bmatrix} \). Find the following, if possible, if an operation is not possible, state why it is not possible.
   (a) \( AB \)    (b) \( BA \)    (c) \( 2B - C \)
9. (a) Find the system of equations that is equivalent to \[
\begin{bmatrix}
2 & 7 \\
1 & 4
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
3 \\
2
\end{bmatrix}. \] (Do not solve the system)

(b) Solve the system of equations in (a) given that the inverse of \[
\begin{bmatrix}
2 & 7 \\
1 & 4
\end{bmatrix}
\] is \[
\begin{bmatrix}
4 & -7 \\
-1 & 2
\end{bmatrix}. \]

10. Use the binomial theorem or Pascal’s triangle to expand \((x^2 - 2y)^5\) (write down Pascal’s triangle if and as far as you use it).

11. Use the binomial theorem to find the term of \((2x - y^3)^{14}\) that contains \(x^{11}\).
12. Solve the system of equations

\[
\begin{align*}
    x + 2y - 3z &= -7 \\
    2x - y + 4z &= 11 \\
    4x + 3y - 4z &= -3
\end{align*}
\]

13. Find the inverse of \( A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 3 & 3 & 7 \end{bmatrix} \). Show all steps.