Math 121, Test 1, October 8, 2008

Instructions. Do each of the following 13 problems. Each problem is worth 5pts. You must show all appropriate details in your solutions. Give exact answers unless you are asked to round. You may use a calculator. Cell phones must be off, and cannot be used for calculations. Good luck.

1. Solve the absolute value equation \(3|x - 3| + 3 = 33\).

\[
3|x - 3| + 3 = 33 \\
3|x - 3| = 30 \\
|x - 3| = 10 \\
\begin{align*}
&x - 3 = -10 \\
&\text{or} \\
&x - 3 = 10 \\
\end{align*}
\]

\[
x = -7 \\
\text{or} \\
x = 13
\]

2. Solve the formula \(0 = \frac{9}{5}C + 32 - F\) for \(C\).

\[
\frac{9}{5}C = F - 32 \\
C = \frac{5}{9}(F - 32)
\]

3. Solve the equation \(x^2 = -8x - 15\).

\[
x^2 + 8x + 15 = 0 \\
(x + 3)(x + 5) = 0 \\
\begin{align*}
&x = -3 \\
&\text{or} \\
&x = -5
\end{align*}
\]

4. Solve the equation \(x^{10} - x^5 - 12 = 0\).

\[
u^2 - u - 12 = 0 \quad \text{where} \quad u = x^5 \\
(u - 4)(u + 3) = 0 \\
u = 4 \quad \text{or} \quad u = -3 \\
x^5 = 4 \quad \text{or} \quad x^5 = -3 \\
\begin{align*}
&x = \sqrt[5]{4} \\
&\text{or} \\
&x = \sqrt[5]{-3}
\end{align*}
\]
5. Use the quadratic formula to solve $3x^2 - 8x + 1 = 0$.

\[
x^2 = \frac{8 \pm \sqrt{64 - 4(3)(1)}}{6}
\]

\[
= \frac{8 \pm \sqrt{52}}{6}
\]

\[
= \frac{8 \pm 2\sqrt{13}}{6}
\]

\[
= \frac{4 \pm \sqrt{13}}{3}
\]

\[
x = \frac{4 + \sqrt{13}}{3} \quad \text{or} \quad x = \frac{4 - \sqrt{13}}{3}
\]

6. Solve the inequality $\frac{x(x + 2)}{x - 3} < 0$. Write your answer in interval notation.

\[\begin{array}{c|c|c|c}
& x + 2 & \quad x & x - 3 \\
\hline
x & - & - & - & 0 & + & + \\
\hline
x - 3 & & - & 0 & + & + & + \\
\hline
\hline
\frac{x(x+2)}{x-3} & - & 0 & + & 0 & - & \frac{3}{3} \text{ und} & + \\
\end{array}\]

\[\left( -\infty, -2 \right) \cup \left( 0, 3 \right)\]

7. Solve the inequality $28 - 3x \leq x^2$. Write your answer in interval notation.

\[\Rightarrow \quad x^2 + 3x - 28 \geq 0\]

\[\Rightarrow \quad (x + 7)(x - 4) \geq 0\]

\[\begin{array}{c|c|c|c|c|c|c|c}
& x + 7 & \quad x & \quad x-4 \\
\hline
x + 7 & 0 & + & + & + & + & + & + \\
\hline
x - 4 & & - & - & - & 0 & + & + \\
\hline
\hline
\quad -1 & 0 & 1 & 4 & 7 & 11 & \infty & + \\
\end{array}\]

\[\left( -\infty, -7 \right] \cup \left[ 4, \infty \right)\]
8. Solve the radical equation \( x + \sqrt{3-x} + 3 = 0 \). Check all proposed solutions.

\[
\begin{align*}
\Rightarrow & \quad x + 3 = -\sqrt{3-x} \\
\Rightarrow & \quad (x+3)^2 = (-\sqrt{3-x})^2 \\
\Rightarrow & \quad x^2 + 6x + 9 = 3 - x \\
\Rightarrow & \quad x^2 + 7x + 6 = 0 \\
\Rightarrow & \quad (x+6)(x+1) = 0 \\
\Rightarrow & \quad x = -6 \quad \text{or} \quad x = -1 \\
\text{(proposed)} & \\
\end{align*}
\]

Check:

\[
\begin{align*}
x = -6: & \quad -6 + \sqrt{3-(-6)} + 3 \\
& \quad = -6 + 3 + 3 = 0 \quad \checkmark \\
x = -1: & \quad -1 + \sqrt{3-(-1)} + 3 = 0 \\
& \quad = -1 + 2 + 3 = 0 \quad \times
\end{align*}
\]

\( x = -6 \)

9. Write the complex number \( \frac{4 - i}{3 + 2i} \) in standard form.

\[
\frac{(4 - i)(3 - 2i)}{(3 + 2i)(3 - 2i)} = \frac{12 - 11i + 2i^2}{9 + 4} = \frac{10 - 11i}{13} = \frac{10}{13} - \frac{11}{13}i
\]

10. In this class, midterms are worth 55% of the final grade, assignments are worth 15% of the final grade and the final test is worth 30% of the final grade. If a student has an average of 79% on midterms and 83% on assignments, what percentage does the student need on the final test to obtain an overall percentage of 75%? Round answer to nearest tenth.

\[
\text{Solve:} \quad 0.55(79\%) + 0.15(83\%) + 0.3x = 75\%
\]

\[
\Rightarrow \quad 0.3x = 19.1\%
\]

\[
\Rightarrow \quad x = 63.7\%
\]

The student needs 63.7% on the final test.
11. A total of $6500 is deposited into two simple interest accounts. On one account the annual interest rate is 4% and on the second account the annual interest rate is 6%. The amount of interest earned in one year is $304. How much was invested in each account?

\[
x = \text{amount @ 6%}
\]
\[
6500 - x = \quad \text{4%}
\]

\[
.06x + (6500 - x)(.04) = 304
\]
\[
\Rightarrow \quad .02x + 260 = 304
\]
\[
\Rightarrow \quad .02x = 44
\]
\[
\Rightarrow \quad x = \$2200
\]
\[
6500 - x = \$4300
\]

\[
\text{\$2200 @ 6\% and \$4300 @ 4\%}
\]

12. The maximum height of a projectile launched into the air varies directly as the square of the time it is in the air. Suppose a projectile that is in the air for 6 seconds reaches a maximum height of 144 feet. What is the maximum height reached by a projectile that is in the air for 8 seconds?

Let Max height = M, time = t

\[
M = k \cdot t^2
\]

\[
144 = k \cdot 36
\]

\[
\Rightarrow \quad k = 4
\]

13. The Ideal Gas Law says that the pressure \( P \) of a gas varies jointly as the number of moles of gas \( n \) and the absolute temperature \( T \) and inversely as the volume \( V \).

(a) Write an equation that represents this relationship.

\[
P = \frac{knT}{V}
\]

(b) What happens to \( P \) if \( V \) is doubled and \( T \) is tripled?

\[
P = \frac{kn(3T)}{2V} = \frac{3}{2} \cdot \frac{knT}{V} = \frac{3}{2} \times \text{original pressure.}
\]

Pressure becomes \( 1.5 \) times as great as original.