1. Find all real solutions to the equation $3x^4 - 22x^2 = 121$.

**Solution:** First write the equation in standard form as $3x^4 - 22x^2 - 121 = 0$, and observe it is an equation of quadratic type. We let $u = x^2$, and then

$$3u^2 - 22u - 121 = 0$$

Then factoring the left side we find $(3u + 11)(u - 11) = 0$, and so $u = 11$ or $u = -\frac{11}{3}$. Because $u = x^2$, this implies $x^2 = 11$ or $x^2 = -\frac{11}{3}$. The equation $x^2 = 11$ implies $x = \pm\sqrt{11}$, but $x^2 = -\frac{11}{3}$ has no real solutions. Therefore, $-\sqrt{11}, \sqrt{11}$ are the real solutions to the equation.

2. Use algebra to solve the equation $x - \sqrt{x - 5} = 7$. Check all proposed solutions.

**Solution:** We rewrite the equation as

$$x - 7 = \sqrt{x - 5}$$

and square both sides to obtain

$$(x - 7)^2 = (\sqrt{x - 5})^2 \quad \text{or} \quad x^2 - 14x + 49 = x - 5$$

and then $x^2 - 15x + 54 = 0$ and factoring implies $(x - 6)(x - 9) = 0$.

Thus the proposed solutions are $x = 6$ and $x = 9$, which we now check by plugging them into the original equation.

$x = 6$: $6 - \sqrt{6 - 5} \neq 7$, and so 6 is not a solution.

$x = 9$: $9 - \sqrt{9 - 5} = 7$, and so $x = 9$ is the solution.
3. A car and a truck are making a long trip on the same route. The car travels at a constant rate 8 km/h faster than the truck that also travels at a constant rate. The truck started the trip half-an-hour before the car. The car overtook the truck after traveling 624 km. What is the rate of each vehicle?

**Solution:** Let \( x \) be the rate of the truck; then \( x + 8 \) is the rate of the car. Using the equation \( t = \frac{d}{r} \) (time is distance over rate), we get

\[
\frac{624}{x + 8} + \frac{1}{2} = \frac{624}{x}
\]

(the time the car travelled plus one-half is the time the truck travelled). Multiplying both sides by \( 2x(x + 8) \) yields

\[
1248x + x^2 + 8x = 1248x + 9984
\]

\[\Rightarrow x^2 + 8x - 9984 = 0\]

Thus,

\[
x^2 + 8x - 9984 = (x - 96)(x + 104) = 0
\]

Therefore, \( x = 96 \) (the rate of the truck is positive). The rate of the truck is 96 km/h and the rate of the car is 104 km/h (since it is 8 km/h more than the rate of the truck).

4. Solve the following inequality

\[-6(x + 2) + 20 \geq 3 - x\]

(a) Express your solution in interval notation.
(b) Graph your solution on a number line.

**Solution:** (a) Using properties of inequalities we find

\[-6(x + 2) + 20 \geq 3 - x \quad \Rightarrow \quad -6x - 12 + 20 \geq 3 - x\]
\[\Rightarrow -6x + 8 \geq 3 - x\]
\[\Rightarrow -6x + 8 + x \geq 3\]
\[\Rightarrow -5x \geq 3 - 8\]
\[\Rightarrow -5x \geq -5\]
\[\Rightarrow x \leq \frac{-5}{-5}\]
\[\Rightarrow x \leq 1\]

Thus, the solution set is \((-\infty, 1]\)

(b) [Graph of the solution set]
5. Solve the inequality \( \frac{(x + 2)(x - 10)}{x + 7} \leq 0 \) using the method of critical values with test numbers or a sign chart. Write the solution set in interval notation.

Solution: The critical values are \( x = -7, \ x = -2 \) and \( x = 10 \). We look for when the expression is negative or zero. Note it is 0 when \( x = -2 \) or \( x = 10 \), and it is undefined when \( x = -7 \), thus the critical values in the solution set are \( x = -2 \) and \( x = 10 \). For the rest we use the sign chart:

<table>
<thead>
<tr>
<th>( x + 7 )</th>
<th>( x + 2 )</th>
<th>( x - 10 )</th>
<th>( \frac{(x + 2)(x - 10)}{x + 7} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>neg</td>
<td>pos</td>
<td>neg</td>
<td>pos</td>
</tr>
<tr>
<td>pos</td>
<td>neg</td>
<td>pos</td>
<td>pos</td>
</tr>
</tbody>
</table>

Thus the answer is: \( (-\infty, -7) \cup [-2, 10] \)

6. (A physics inequality) The equation \( s = -16t^2 + v_0 t + s_0 \) gives the height \( s \), in feet above ground level, at the time \( t \) seconds, of a projectile launched directly upward from a height \( s_0 \) feet above the ground and with an initial velocity of \( v_0 \) feet per second. A catapult launches a rock directly upward from an initial a height of 11 feet above the ground with an initial velocity of 208 feet per second. Find the time interval during which the rock will be more than 363 feet above the ground.

Hint. This will reduce to a quadratic inequality where all terms have a factor of 16.

Solution: The problem suggest \( v_0 = 208 \) and \( s_0 = 11 \). Thus \( s = -16t^2 + 208t + 11 \). We then solve the inequality \( -16t^2 + 208t + 11 > 363 \). Thus \( -16t^2 + 208t - 352 > 0 \) and multiplying both sides by \(-1\) we have \( 16t^2 - 208t + 352 < 0 \) and dividing by \(16\) (noted in the hint) yields \( t^2 - 13t + 22 < 0 \) and factoring gives us \( (t - 2)(t - 11) < 0 \). The critical values are 2 and 11, and the following sign chart helps us solve the inequality:

<table>
<thead>
<tr>
<th>( t - 2 )</th>
<th>( t - 11 )</th>
<th>( (t - 2)(t - 11) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>neg</td>
<td>neg</td>
<td>pos</td>
</tr>
<tr>
<td>0</td>
<td>neg</td>
<td>neg</td>
</tr>
<tr>
<td>2 &lt; t &lt; 11</td>
<td>t = 11</td>
<td>11 &lt; t</td>
</tr>
<tr>
<td>pos</td>
<td>0</td>
<td>pos</td>
</tr>
</tbody>
</table>

Thus the inequality is true when \( 2 < t < 11 \). That is the rock is more than 363 feet above the ground between 2 seconds and 11 seconds after it was launched.
7. The area, \( A \), of a picture projected on a movie screen varies directly as the square of the distance, \( d \), from the projector to the screen.

(a) Write an equation that expresses the relationship between the variables. Use \( k \) as the constant of variation.

(b) If a distance of 20 feet produces a picture with an area of 58 square feet, what distance produces a picture with an area of 1450 square feet.

**Solution:** (a) The relation is \( A = kd^2 \).

(b) For the projector in (b), we have \( 58 = k(20)^2 \) and so \( k = \frac{58}{(20)^2} \). Then, to produce an area of 1450 square feet, we need

\[
1450 = \frac{58}{(20)^2} d^2 \quad \Rightarrow \quad d^2 = \frac{1450 \cdot (20)^2}{58}
\]

so \( d^2 = 25(20)^2 \) which implies \( d = 5(20) = 100 \) feet.

8. (a) The load \( L \) a horizontal beam can safely support varies jointly as the width \( w \) and the square of the depth \( d \) and inversely as the length \( l \). Write an equation that represents the relation between the given variables. Use \( k \) as the constant of variation.

(b) If a 15-foot beam with width 9 inches and depth 10 inches safely supports 850 pounds, how many pounds can a 19-foot beam that has width 9 inches and depth 9 inches be expected to support? Round answer to the nearest pound. Assume the two beams are made of the same material.

**Solution:** (a) The relation is \( L = \frac{kwd^2}{l} \).

(b) We use \( 850 = \frac{k(9)(10^2)}{15} \) to find \( k = \frac{(15)(850)}{(9)(10^2)} \approx 14.16666667 \). Thus the new beam can support

\[
L \approx \frac{14.1666667(9)(9^2)}{19} \approx 543.553
\]

Thus the new beam can support approximately 544 pounds.