Name: ________________________________

Instructions. Do each of the following 12 problems. Each problem is worth 5pts. Show all appropriate details in your solutions. No calculators are not allowed on the first 9 problems.

1. (a) Find the midpoint of (3, −2) and (7, −4).

(b) Find the distance between (3, −2) and (7, −4).

2. A circle is centered at (−1, 2) and passes through the point (2, −6). Find the radius of the circle, and then write the equation of the circle.

3. (a) Find \( f(1) \), \( f(5) \) and \( f(7) \) for the piecewise defined function \( f(x) = \begin{cases} \frac{x^3 + 2}{1-x} \quad & \text{if } x < 1; \\ 0 \quad & \text{if } 1 \leq x \leq 5; \\ x - 2 \quad & \text{if } x > 5. \end{cases} \)

(b) Find the domain of \( f(x) = \sqrt{7-x} \) and state your answer in interval form.
4. Find the equation of the line through the points (4, 5) and (7, −1). Write your answer in slope-intercept form.

5. Let $f(x) = ax^2 + bx + c$ be a quadratic where $a \neq 0$.
   (a) If $a > 0$, does the function $f$ have a minimum or a maximum?
   (b) What is the formula for finding the vertex of $f$?
   (c) What is the range of the function $h(x) = -4(x + 2)^2 - 3$?

6. Use the graph of $g$ to sketch (a) $y = g(x + 3)$ and (b) $y = g(x + 3) - 2$ and (c) $y = g(-x)$. 
7. (a) Consider the function \( f(x) = 6 - 2|x| \). Determine whether \( f \) is an even function, odd function or neither.

(b) What are the \( x \)-intercepts of \( y = 6 - 2|x| \)?

(c) What are the \( y \)-intercepts of \( y = 6 - 2|x| \)?

8. Let \( f(x) = \sqrt{x - 4} \) and \( g(x) = -3x \).

(a) What is the domain of \( \frac{f}{g} \)?

(b) Find \((f + g)(5)\), \((f - g)(5)\), \((fg)(5)\) and \((f \circ g)(-5)\).

9. Find the difference quotient \( \frac{f(x+h) - f(x)}{h} \) of \( f(x) = -x^2 + 3x \).
10. A manufacturer produces a product at a cost of $22.80 per unit. The manufacturer has a fixed cost of $400.00 per day. Each unit retails for $37.00. Let $x$ be the number of units produced in a day.

(a) Write the cost $C$ as a function of $x$.

(b) Write the revenue $R$ as a function of $x$.

(c) Write the profit $P$ as a function of $x$ (remember: profit = revenue - cost).

11. The number of board-feet of lumber that can be obtained from a log 16 feet long is linearly related to diameter as follows. If the diameter is 16 inches, then 90 board-feet are obtained; if the diameter is 22 inches, then 180 board-feet are obtained.

(a) Use this data to find the formula of a linear function $f(x)$ which gives the number of board-feet of a 16 foot log when $x$ is the diameter in inches of the log.

(b) Use $f$ to determine how many board-feet a log 16 feet long and 2 feet in diameter would produce?

12. A farmer has 1200 feet of fencing to build a rectangular pen that is subdivided into two separate rectangular regions for his pigs and dogs (see sketch below).

(a) Write the length $l$ as a function of the width $w$.

(b) Write the total area $A$ as a function of $w$.

(c) Find the dimensions that produce the greatest enclosed area.