Name: Hints or Answers

Instructions. Do each of the following eleven questions. Show all of your work. Do not use a calculator on the first part. Good luck.

1. Use synthetic division to find \((5x^5 + 10x^4 - 5x + 2) \div (x + 2)\).

\[
\begin{array}{c|ccccc}
-2 & 5 & 10 & 0 & 0 & -5 \\
 & & -10 & 0 & 0 & 10 \\
\hline 
 & 5 & 0 & 0 & 0 & -5 \\
\end{array}
\]

Ans: \(5x^4 - 5 + \frac{12}{x+2}\)

2. Let \(P(x) = x^{101} - 3x + 2\).
   
   (a) What is the remainder of \(P(x) \div (x + 1)\)? Is \((x + 1)\) a factor of \(P(x)\)?
   
   \[R = P(-1) = (-1)^{101} - 3(-1) + 2 = -1 + 3 + 2 = 4\]  
   
   Since \(R \neq 0\), \((x+1)\) is not a factor of \(P(x)\).

   (b) What is the remainder of \(P(x) \div (x - 1)\)? Is \((x - 1)\) a factor of \(P(x)\)?
   
   \[R = P(1) = (1)^{101} - 3 + 2 = 1 - 3 + 2 = 0\]  
   
   Since \(P(1) = 0\), the factor theorem says \((x-1)\) is a factor of \(P(x)\).

3. Use the Rational Zero Theorem to list all possible rational zeros of \(P(x) = 4x^5 + 11x^2 - 6x - 6\).

   \[\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}\]

4. The length of a rectangular box is 1 inch more than twice the height of the box, and the width is 3 inches more than the height.

   (a) Find a formula for volume \(V(x)\) where \(x\) is height in inches.

   \[l = 2x + 1\]  
   
   \[w = x + 3\]  
   
   \[h = x\]  

   \[V(x) = x(x + 3)(2x + 1) = x(2x^2 + 7x + 3) = 2x^3 + 7x^2 + 3x\]

   (b) If the volume of the box is 126 cubic inches, write a polynomial whose zero(s) could be used to help find the height of the box (you do not need to find the zeros of the polynomial).

   Solve \(V(x) = 126\)  

   \[2x^3 + 7x^2 + 3x = 126\]  

   \[2x^2 + 7x + 3 - 126 = 0\]

   \[x \text{ is a zero of } P(x) = 2x^3 + 7x^2 + 3x - 126\]
5. Let \( P(x) = -3(x+4)^3(x-1)^2(x-3)^2 \). Determine the zeros of \( P(x) \) with their multiplicities. At each zero of \( P(x) \) determine whether the graph of \( y = P(x) \) crosses the \( x \)-axis or merely touches the \( x \)-axis.

<table>
<thead>
<tr>
<th>zero</th>
<th>multiplicity</th>
<th>cross or merely touch</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>3</td>
<td>crosses, not cross</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>crosses</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>touched, does not cross</td>
</tr>
</tbody>
</table>

6. Given \( P(x) = -3(x+4)^3(x+1)^4(x-3)^2 \) as in the previous question. Determine the degree of \( P \). Determine the far right and far left behavior of \( P \). Using this along with the information you found in the previous question, sketch a rough graph of \( P \).

\[
\text{Deg } P = 9 \\
\text{a}_9 = -3 < 0 \\
\text{up to far left} \\
\text{down to far right}
\]

7. Find all zeros of \( P(x) = x^4 - 4x^3 + 14x^2 - 4x + 13 \) given that \( i \) is a zero of \( P(x) \). Then write \( P(x) \) as a product of its linear factors.

\[
\begin{array}{cccccc}
& 1 & -4 & 14 & -4 & 13 \\
\text{i} & -1 & 4i & 9 + 3 & -3 & 0 \\
-\text{i} & 1 & -4 + i & 9 - 3 & 3 & 0 \\
& 1 & -4 & 13 & 0 \\
\end{array}
\Rightarrow x = 2 \pm 3i
\]

\[
P(x) = (x - i)(x + i)(x - 2 - 3i)(x - 2 + 3i)
\]

8. Use Descartes' Rule of signs to state the number of possible positive and negative real zeros of \( P(x) = 3x^5 - 2x^4 - x^3 + 7x^2 + 12x - 5 \).

\[
P(-x) = -3x^5 - 2x^4 + x^3 + 7x^2 + 12x - 5
\]

3 or 1 positive real zeros
2 or 0 negative real zeros
9. Find the polynomial of degree 3 that has zeros \(3i, 2\) and such that \(P(3) = 36\).

\[
P(x) = a_3 \, (x-2)(x-3i)(x+3i) \\
= a_3 \, (x-2)(x^2 + 9) \\
P(3) = 36 \Rightarrow a_3 \cdot 1 \cdot 12 = 36 \\
\Rightarrow a_3 = 2
\]

\[
P(x) = 2(x-2)(x^2 + 9) \\
= 2(x^3 - 2x^2 + 9x - 18) \\
= 2x^3 - 4x^2 + 18x - 36
\]

10. Given \(G(x) = \frac{x^2 - x}{x + 2}\). Find and write the equations for all horizontal, vertical and slant asymptotes for \(G\). If a certain type of asymptote does not exist, make sure to say so.

i) No horizontal asymptote

ii) \(x = -2\) is vertical asymptote

iii) \[\begin{array}{c|c|c}
-2 & 1 & 0 \\
-1 & 3 & 6 \\
\end{array}\]

\[
y = x - 3 \quad \text{is slant asymptote}
\]

11. For \(G\) as in the previous question. Find the \(x\)-intercepts and \(y\)-intercepts for the graph \(y = G(x)\), complete the following table, and then graph \(y = G(x)\) along with its asymptotes.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-4</th>
<th>-3</th>
<th>-2.01</th>
<th>-1.99</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G(x))</td>
<td>-10</td>
<td>-12</td>
<td>-605.01</td>
<td>595.01</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

\(x\)-intercept(s): \((0,0), (1,0)\) \[x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, \ x = 1]\n
\(y\)-intercept: \(G(0) = \frac{0}{2} = 0 \Rightarrow (0,0)\) is \(y\)-intercept.