1. Write the equation of the conic \( x^2 + 6x - 8y - 23 = 0 \) in standard form, and then find its vertex, focus and the equation of its directrix. (Think about similar questions for ellipses and hyperbolas where you would find center, vertices, foci, and additionally asymptotes for hyperbolas)

2. (a) Find the equation of the hyperbola with foci \((2, -6)\) and \((2, 4)\) whose vertices are \((2, -4)\) and \((2, 2)\); and find the equation of its asymptotes.
(b) Find the equation of ellipse with vertices \((-6, 2)\) and \((4, 2)\) and foci \((-4, 2)\) and \((2, 2)\).
(c) Sketch the conics you found in (a) and (b). (Think about analogous questions for parabolas where one is given, e.g. the focus and vertex).

3. (a) Describe the graph whose polar equation is \( r \sin \theta = 3 \). Be specific!
(b) Describe the graph whose polar equation is \( r = 4 \). Be specific!
(c) Convert the point \((-2, 2\sqrt{3})\) to polar coordinate form \((r, \theta)\):
(d) Convert the point \((-2, 5)\) to polar coordinate form, express \( r \) exactly, and approximate \( \theta \) to the nearest \(0.1^\circ\).
(e) Convert the polar point \((-6, -120^\circ)\) to rectangular coordinates:
(f) Convert the polar equation \( r = 6 \cos \theta \) to rectangular form. Describe the graph of the equation.

4. Consider the conic whose equation in polar coordinates is \( r = \frac{9}{3 - 6 \sin \theta} \).
(a) What is its eccentricity?
(b) What type of conic is it?
(c) Convert the equation \( r = \frac{9}{3 - 6 \sin \theta} \) to rectangular coordinates

5. (a) Eliminate the parameter for the curve given by the parametric equations \( x = 2 - t, y = 1 - t^2 \) for \(-\infty < t < \infty\) to find its equation in rectangular coordinates.
(b) Sketch the curve given by \( x = 2 - t, y = 1 - t^2 \) where \(-2 \leq t \leq 1\).
(c) Eliminate the parameter and sketch the curve given by the parametric equations \( x = -3 + 4 \cos t, y = 2 - 3 \sin t \) for \( 0 \leq t \leq 2\pi \). Indicate the direction of motion on the curve.
(d) Sketch the curve given by \( x = -3 + 4 \cos t, y = 2 - 3 \sin t \) for \( 0 \leq t \leq \pi \).

6. (a) Given a conic \( r = \frac{ed}{1 - e \sin \theta} \) where \( d > 0 \). Describe what type of conic it is based on the value of \( e \), and explain whether the directrix is horizontal or vertical and where it is in relation to the pole.
(b) Find the polar equation of an ellipse with focus at the pole, and vertices \((6, 0)\) and \((2, \pi)\). Describe the directrix of this conic. (Analogous questions for parabolas and hyperbolas with focus at pole and vertex/vertices given)
(c) Sketch the conic given by \( r = \frac{3}{1 - 2 \cos \theta} \). Confirm your answer by converting the equation to rectangular form.
In addition to questions as on the previous page, some of the key calculus operations are:

**Parametric Curves.** Computing $\frac{dy}{dx}$ and higher derivatives of parametric curves. Finding tangent lines, determining concavity of parametric curves. Finding the length of a smooth parametric curve.

**Polar Curves.** Computing $\frac{dy}{dx}$ of polar curves, and finding tangent lines to the curves. Finding the arc length of a polar curve, and finding areas enclosed by, enclosed between, etc of polar curves.

Some sample questions of in these topics are as follows.

7. Let $x = 5 + 2 \cos t$ and $y = 3 + 4 \sin t$.
   (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for this curve.
   (b) Find the tangent line to the curve when $t = \pi/6$.
   (c) If the graph concave up, or concave down when $t = \pi/6$?

8. Set-up an integral to find the perimeter of the curve given in 7. (What type of curve is it?)

9. Compute arc length of the curve $x = e^{-t} \cos t$ and $y = e^{-t} \sin t$ for $0 \leq t \leq \pi/2$; see p. 728 # 53.

10. (a) Sketch the region that lies inside the curve $r = 4 + 4 \sin \theta$ and outside the curve $r = 6$.
    (b) Find the area of the region described in (a).
    (c) Find $\frac{dy}{dx}$ for the polar curve $r = 4 + 4 \sin \theta$; then find the line tangent to this curve at the point $(6, 5\pi/6)$

11. Find the length of the polar curve $r = 2 - 2 \cos \theta$. (Be able to set-up the integral, the evaluation requires a nice application of trig identities—see Example 4, p. 745).

12. Find the equation in polar coordinates of a parabola with focus at the pole and vertex at $(2, \pi)$. Then set-up and integral to find the length of this curve between the polar points $(4, \pi/2)$ and $(4, 3\pi/2)$. 