Name: Answers and Hints

1. (a) Solve \( R(2 - 1.2F) = 0 \) and \( -F(1 - .9R) = 0 \) The first equation implies \( R = 0 \) or \( F = 5/3 \).

If \( R = 0 \), we plug this into the second equation to get \( F = 0 \). Thus \((0, 0)\) is an equilibrium point.

If \( F = 5/3 \), we plug this into the second equation to get \( R = 10/9 \). Thus the equilibrium points of the system are:

\((0, 0)\) and \((10/9, 5/3)\).

(b) Increase \( .9 \) significantly in the equation \( \frac{dF}{dt} = -F + .9RF \).

(c) \( \frac{dR}{dt} = 2R(1 - R/50) - 1.2RF \), \( \frac{dF}{dt} \) remains the same.

(d) \( \frac{dF}{dt} = -F + .9RF + \alpha R \) where \( \alpha > 0 \) is a constant.

(e) Compute \( \frac{dR}{dt} = -1.2 \) and \( \frac{dF}{dt} = 3.4 \) at \((3, 2)\) and so the prey population is decreasing while the predator population is increasing.

2. The table for Euler’s method is as follows

<table>
<thead>
<tr>
<th>( i )</th>
<th>( x_i )</th>
<th>( y_i )</th>
<th>( m_i )</th>
<th>( n_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>1 + .1(1) = 1.1</td>
<td>1 + .1(-1) = .9</td>
<td>.9</td>
<td>-1.289</td>
</tr>
<tr>
<td>2</td>
<td>1.1 + (.1)(.9) = 1.19</td>
<td>.9 + (1.1)(-1.289) = .7711</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus, the solution is approximately \((1.19, .7711)\) when \( t = .2 \)

3. (a) Let \( v = \frac{dy}{dt} \) and then \( \frac{dv}{dt} = -3v + 10y \). Now with

\[
Y = \begin{bmatrix} y \\ v \end{bmatrix}, \quad \frac{dY}{dt} = \begin{bmatrix} 0 & 1 \\ 10 & -3 \end{bmatrix} Y.
\]

(b) Pug \( y = e^{mt} \) into the differential equation to get \( m^2 e^{mt} + 3m e^{mt} - 10 e^{mt} = 0 \), and thus \( m^2 + 3m - 10 = 0 \) and so \( m = -5 \) and \( m = 2 \).
Thus solutions are $y = e^{-5t}$ or $y = e^{2t}$.

(c) Solve $\begin{vmatrix} -\lambda & 1 \\ 10 & -3 - \lambda \end{vmatrix} = 0$. Thus $\lambda^2 + 3\lambda - 10 = 0$ and so $\lambda = -5$ and $\lambda = 2$. These are the same values as we found for $m$ in (b), which they should be.

4. To verify the eigenvalues and eigenvectors, multiply the matrix by the eigenvectors.

$$\begin{bmatrix} 3 & 4 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-1) \begin{bmatrix} 1 \\ -1 \end{bmatrix}. $$

thus $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector corresponding to $\lambda = -1$. Now

$$\begin{bmatrix} 3 & 4 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 4 \\ 1 \end{bmatrix}. $$

thus $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ is an eigenvector corresponding to $\lambda = 4$. Now

(b) The general solution to the system is in vector form

$$Y = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{4t}. $$

In individual function form, the solution is

$$x(t) = c_1 e^{-t} + 4c_2 e^{4t} \quad y(t) = -c_1 e^{-t} + c_2 e^{4t}. $$

(c) In the solution just written, solve $c_1 + 4c_2 = 6$ and $-c_1 + c_2 = 0$. Thus $c_1 = c_2$ and $5c_1 = 6$. So $c_1 = 6/5$ and $c_2 = 6/5$. Therefore,

$$Y(t) = \frac{6}{5} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + \frac{6}{5} \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{4t}. $$

(d) and (e). The sketches are left to the reader. But notice that the origin is a saddle, also, solutions tend toward the straight line solution $y = \frac{1}{4}x$ as $t$ increases, and they tend toward the straight line solution $y = -x$ as $t$ decreases.