1. (5 pts) Solve the differential equation $\frac{dy}{dt} = ty^2 + 2y^2$ subject to $y(0) = 1$.

2. (5 pts) Use Euler’s method to estimate the solution to $\frac{dy}{dt} = t - 3y$ subject to $y(0) = 1$ at $t = .5$ using $\Delta t = 0.25$.

3. (5 pts) Suppose a 50 gallon barrel contains 10 gallons of water with 2 lbs of salt in it. A saline solution containing 1 lb of salt per gallon is pumped into the barrel at 3 gallons per minute while the well-mixed solution is drained out at 1 gallon per minute. Find, but do not solve, an initial valued differential equation modeling the amount of salt in the barrel.
4. (5 pts) Ken says that \( y = t^3 \) is a solution to the differential equation \( \frac{dy}{dt} = 3y^2 \). Tom says that this can’t be true because \( y = 0 \) is an equilibrium solution, and no solution can cross an equilibrium solution by the uniqueness theorem. Please help Tom and Ken out. Is \( y = t^3 \) a solution? Is \( y = 0 \) an equilibrium solution? Does \( y = t^3 \) cross the equilibrium solution? Is there a problem if it does cross the equilibrium solution? Explain.

5. (5 pts) Draw the phase line for the differential equation \( \frac{dy}{dt} = y^2(y - 2)(y + 3) \). Identify the equilibrium points as sources, sinks or nodes, and describe the long term behavior of a solution to this equation passing through the point (2,-10).

6. (5 pts) The change of variables \( u = y + t \) converts the differential equation \( \frac{dy}{dt} = (y + t)^2 - 4(y + t) + 2 \) into \( \frac{du}{dt} = (u - 3)(u - 1) \). Sketch some graphs of solutions of the original differential equation.
7. (5 pts) Find the general solution to the differential equation \( \frac{dy}{dt} = -2ty + 4e^{-t^2} \).

8. (10 pts) Consider the 1-parameter family of differential equations \( \frac{dP}{dt} = -\frac{P^2}{400} + P - K \) which represents a population of fish, where \( K \geq 0 \) is the parameter for the number of fish that are harvested each year.

(a) (3 pts) Find the bifurcation value(s) of \( K \).

(b) (2 pts) What is the maximum number of fish that can be harvested each year so that the population does not go extinct?

(c) (5 pts) If 36 fish are harvested each year, describe the various long term population trends depending on the initial population. Hint: think of the phase line.