Math 232: Review for Final Test

The final test is at 8:00am, Tuesday, March 16, 2004. The test will include topics from the entire quarter. For review, you should review topics from previous tests. See those tests and the review material supplied for those tests, and see also tests from previous years as available on-line.

The new material since the last test, includes sections 5.1 and 5.2 on nonlinear systems and using Laplace transform to solve differential equations. For review on nonlinear systems, see the assignment questions from sections 5.1 and 5.2. For review on Laplace Transforms, practice doing transforms of simple functions using the definition (e.g. \( L(e^{at}) \), \( L(1) \), or \( L(t) \)), and see the questions below.

1. Solve the following differential equations using Laplace transforms. You may use MathCAD to do the partial fractions.

   (a) \( y'' - 6y' + 9y = t^2e^{3t} \) subject to \( y(0) = 2, \, y'(0) = 6 \).

   (b) \( y'' + 5y' + 4y = 0 \) subject to \( y(0) = 1, \, y'(0) = 0 \).

   (c) \( y'' - 6y' + 9y = t \) subject to \( y(0) = 0, \, y'(0) = 1 \).

   (d) \( y'' + 16y = 1 \) subject to \( y(0) = 1, \, y'(0) = 2 \).

2. In each of the following find a formula for \( L[y] \) where \( y(t) \) is the solution to the initial value problem.

   (a) \( y'' - 2y' + 5y = 1 + t \) subject to \( y(0) = 0, \, y'(0) = 4 \).

   (b) \( y'' + y = \sin t \) subject to \( y(0) = 1, \, y'(0) = -1 \).

   (c) \( y'' - 4y' + 4y = t^3e^{2t} \) subject to \( y(0) = 0, \, y'(0) = 0 \).

3. Given the following Laplace transforms \( L[y] \) where \( y(t) \) is a continuous function. Find \( y(t) \).

   (a) \( L[y] = \frac{7}{25s} + \frac{1}{5s^2} + \frac{-7s/25 + 109/25}{25(s^2 - 2s + 5)} \) Note: this corresponds to 2(a).

   (b) \( L[y] = \frac{s - 1}{s^2 + 1} + \frac{1}{(s^2 + 1)^2} \) Note: this corresponds to 2(b).

   (c) \( L[y] = \frac{6}{(s - 2)^6} \) Note: this corresponds to 2(c).

   (d) Given that the initial value problem \( y'' + 16y = 1 \) subject to \( y(0) = 1, \, y'(0) = 2 \) transforms to \( L[y] = \frac{1}{16s} + \frac{15s/16 + 2}{s^2 + 16} \), find \( y(t) \).