Name: ______________________________

Instructions. Do 8 of the following 9 questions. Each is worth 5 points.

1. Consider the following system of differential equations modeling a predator-prey system.

\[
\frac{dx}{dt} = .5x \left(1 - \frac{x}{2000}\right) + .3xy \\
\frac{dy}{dt} = .7y \left(1 - \frac{y}{1000}\right) - 1.2xy
\]

Identify which variable is the predator and which is the prey. Do the predators have sources of food other than the prey? Explain your answers.

2. Solve the differential equation \( \frac{dy}{dt} = ty^2 + 2y^2 \) subject to \( y(0) = 1 \).

3. Use Euler’s method to estimate the solution to \( \frac{dy}{dt} = t - y^2 \) subject to \( y(0) = 1 \) at \( t = .5 \) using \( \Delta t = 0.25 \).
4. Draw the phase line for the differential equation \( \frac{dy}{dt} = y^2 \left( 1 - \frac{y}{50} \right) \left( \frac{y}{100} - 1 \right) \). Identify the equilibrium points as sources, sinks or nodes. Finally, sketch solutions with the following initial conditions: (i) \( y(0) = 25 \), (ii) \( y(0) = 50 \). (iii) \( y(0) = 75 \), (iv) \( y(0) = 125 \).

5. Once again, Ken and Tom were arguing about solutions to \( \frac{dy}{dt} = 5y^{4/5} \) s.t. \( y(0) = -32 \). Tom said the solution cannot cross \( y = 0 \) because of the uniqueness theorem, whereas Ken said that \( y(t) = (t - 2)^{5} \) is a solution to the problem, and it does cross the \( y = 0 \). (a) Is Tom correct? Explain. (b) Is Ken correct? Explain.

6. It was found that a 1000 gallon tank contained 500 gallons of water containing .25 grams of mercury per gallon. Bob began filling the tank with water containing .01 grams of mercury per gallon at a rate of 10 gallons per minute while he drained the well mixed solution at a rate of 5 gallons per minute. Write an initial valued differential equation to model \( A(t) \) the amount of mercury in the tank at time \( t \). Do not solve the problem.
7. Use the change of variable $u = \frac{y}{1+t}$ to convert the differential equation

$$\frac{dy}{dt} = \frac{y}{1+t} - \frac{y}{t} + t^2(1+t)$$

into the form $\frac{du}{dt} = f(u, t)$. Describe the resulting equation as linear, separable, or neither.

8. Find the general solution to the differential equation in 7.

9. Consider the family of d.e.’s $\frac{dP}{dt} = -\frac{P^2}{1000} + P - 5K$ which represent a population of fish, where $K \geq 0$ is the parameter. Find the bifurcation value(s) of $K$ and draw phase lines at the bifurcation values, and for values of $K$ on either side of the bifurcation value(s).