Name: __________________________

Instructions. Do each of the following six questions. Please show all appropriate work in your solutions in order to obtain maximum credit. You may use a calculator.

1. Solve the differential equation $\frac{dy}{dt} = \frac{t}{y - t^2 y}$ subject to $y(0) = 4$.

2. Use Euler’s method to approximate $y(1.5)$ for the initial valued problem $\frac{dy}{dt} = y^2 - t$ subject to $y(1.3) = 2$. Use $\Delta t = .10$.

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<th>$k$</th>
<th>$t_k$</th>
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Answer: $y(1.5) = \_\_\_\_$
3. Consider the differential equations (i) \( \frac{dy}{dt} = 3(y + 3)^{2/3} \) and (ii) \( \frac{dy}{dt} = (y + 3)^2 \).
(a) Find equilibrium points and draw phase lines for each equation.
(b) For each equation determine whether the uniqueness theorem applies for the equilibrium solution. In particular, for each equation answer whether you can guarantee that no other solution will cross an equilibrium solution.

4. (a) Draw the phase line for the differential equation \( \frac{dP}{dt} = 3P^2 \left( \frac{P}{500} - 1 \right) \left( 1 - \frac{P}{1000} \right) \) that models the population growth for a type of rodent. Identify the equilibrium points as sources, sinks or nodes.
(b) For which initial populations will this population survive long term? Will the population approach a stable number, if so, what will it approach?
(c) For which initial populations will the population go extinct?
5. Consider the one parameter family of differential equations $\frac{dy}{dt} = ky - y^3$. Determine the bifurcation values for $k$, and draw representative phase lines at, and on each side of the bifurcation value(s).

6. An 1800 liter spa initially contains 600 liters of pure water. Water containing 3mg per liter of chlorine is pumped into the spa at a rate of 60 liters per minute, while the well-mixed water is discharged at a rate of 30 liters per minute. Find the amount (in mg) and concentration (in mg/liter) of chlorine in the spa when the spa is full?