Instructions. Do each of the following questions. Please show all of your work. Good Luck!

1. (Short Answers) (a) Suppose $\frac{dY}{dt} = AY$ is a system of differential equations where $A$ has eigenvalues with corresponding eigenvectors: $2, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $-3, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Write the general solution to this system.

(b) Write the second-order differential equation $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = e^t$ as a system of first-order differential equations.

(c) Determine whether $Y(t) = (e^{-t}, 3e^{-t})$ is a solution to the system

$$\frac{dx}{dt} = 2x - y \quad \frac{dy}{dt} = x - 2y.$$ 

(d) Is $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ an eigenvector for the matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$? If so, find its corresponding eigenvalue.

(e) Consider the system $\frac{dx}{dt} = 2x \left(1 - \frac{x}{2}\right) - xy$ and $\frac{dy}{dt} = 3y \left(1 - \frac{y}{3}\right) - 2xy$. Is it a predator-prey system, or a competing species system? Explain.
2. Find all equilibrium points for the system

\[
\frac{dx}{dt} = x(2 - x - y) \quad \frac{dy}{dt} = y(y - x)
\]

3. Find the general solution to the partially decoupled system \( \frac{dx}{dt} = x + y^2 \) and \( \frac{dy}{dt} = 3y \).
4. Use Euler’s method for systems to estimate \((x(2), y(2))\) for

\[
\frac{dx}{dt} = 3y \quad \frac{dy}{dt} = x - y^2
\]

with \(\Delta t = 0.1\) given that \(x(0) = 0\) and \(y(0) = 2\).

\[
\begin{array}{|c|c|c|c|c|}
\hline
i & x_i & y_i & m_i & n_i \\
\hline
\hline
\hline
\end{array}
\]

Answer: \((x(2), y(2)) = \)__________

5. (a) Find the general solution to the system of linear differential equations

\[
\frac{dx}{dt} = -2x - 2y \quad \frac{dy}{dt} = -2x + y
\]

and then (b) find the solution with the initial condition \((x(0), y(0)) = (1, -2)\)
6. Sketch the phase portraits for the linear systems $\frac{dy}{dt} = AY$ where:

(a) $A = \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix}$ with eigenvalues and eigenvectors $-4, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $-1, \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} -2 & -3 \\ -3 & -2 \end{bmatrix}$ with eigenvalues and eigenvectors $-5, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$