Math 233: Test II, Spring 2007 – Multiple Integrals

Name: __________________________

Instructions. Do question 1 and any 4 of questions 2 through 6. Please justify all answers and do not use a calculator.

1. (15 pts) (a) Sketch the region of integration for \( \int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} y \, dy \, dx \).

(b) Use geometry to determine value of integral \( \int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} 1 \, dy \, dx \). (Same limits as (a))

(c) Write the double integral from (a) using polar coordinates.

(d) The integral given in cylindrical coordinates as \( \int_{0}^{2\pi} \int_{0}^{2} \int_{0}^{10} rdz \, dr \, d\theta \) represents a volume of a solid. Describe the solid.

(e) Set-up but do not evaluate an integral in spherical coordinates find the volume of the solid ball of radius 4 centered at the origin.
2. (10 pts) Evaluate the integral \( \int_0^4 \int_{x/2}^2 e^{y^2} \, dy \, dx \) by reversing the order of integration.

3. (10 pts) Consider the planar lamina bounded by \( x = 2 - y^2 \) and \( y = x \) whose density is given by \( \rho = ky^2 \).

(a) Write the formulas for \( \bar{x} \) and \( \bar{y} \) in terms of \( M, M_x \) and \( M_y \).

(b) Write the integrals needed to find \( M, M_x \) and \( M_y \) for this lamina, but do not evaluate the integrals.
4. (10 pts) Use integration to find the surface area of the portion of the cone $z = 3\sqrt{x^2 + y^2}$ that lies between the planes $z = 3$ and $z = 12$.

5. (10 pts) Evaluate $\int \int_{R} (x - y)(x + 4y)^2 dA$ where $R$ is the parallelogram with vertices $(0, 0)$, $(2, 2)$, $(6, 1)$ and $(4, -1)$.

Hint: You may wish to do this as a double integral and then use the change of variables $u = x - y$ and $v = x + 4y$. 
6. (a) (5 pts) Evaluate the integral \( \int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{0}^{5} x \, dz \, dx \, dy \); use any method you wish.

(b) (5 pts) Rewrite the integral \( \int_{0}^{4} \int_{z}^{4} \int_{0}^{\sqrt{y^2-z^2}} xyz \, dx \, dy \, dz \) using the order \( dz \, dy \, dx \). Do not evaluate the integral.