1. Consider the following population consisting of 25 numbers.

\[
\begin{array}{cccccccccccc}
20 & 23 & 24 & 25 & 26 & 30 & 30 & 31 & 32 & 32 & 32 & 33 \\
35 & 39 & 40 & 41 & 41 & 48 & 54 & 54 & 58 & 60 & 62 & 72 
\end{array}
\]

(a) Given that \( \sum x = 966 \) and \( \sum x^2 = 42,140 \) find the mean, variance and standard deviation for this population.

(b) Repeat (a), but assume the data is from a sample.

**Answer.** (a) This is a population, so \( \mu = \frac{966}{25} = 38.64; \)

\[
\sigma^2 = \frac{42140 - \frac{966^2}{25}}{25} = 192.5504.
\]

Thus \( \sigma = \sqrt{192.5504} \approx 13.87625. \)

(b) If the data were only from a sample, then \( n = 25, \) the variance is

\[
s^2 = \frac{42140 - \frac{966^2}{25}}{24} \approx 200.5733
\]

Thus the sample standard deviation is \( s \approx \sqrt{200.5733} \approx 14.162. \)

2. A population is known to have a mean of 150 and a standard deviation of 12.

(a) Use Chebyshev’s theorem to find an interval that contains at least \( 8/9 \) of the data.

(b) At least what portion of data is contained in the interval from 102 to 198?

**Answer.** (a) \( 8/9 = 1 - 1/3^2, \) and so the interval is from \( \mu - 3\sigma = 150 - 3(12) \) to \( \mu + 3\sigma = 150 + 3(12), \) thus the interval is from 114 to 186.

(b) 102 = 150−4(12) and 198 = 150+4(12), so this interval contains at least \( 1-1/4^2 = 15/16 \)th of the data, i.e. at least 93.75% of the data.