In-class Exercises on Probability, 19 October 2007

Name.  

Hints and Answers

1. (a) A pizza shop offers a lunch special that includes soup, salad, small pizza, drink and dessert. If there are 3 types of soup, 2 types of salad, 10 types of pizza, 6 types of drink and 4 choices for dessert, how many possible lunch combinations are there?

(b) Suppose Abe, Bart, Carol, and Denise compete in a math competition, how many ways can prizes of $5000 and $1000 be awarded to the top two finishers? Check by listing all possible ways to award first and second prize.

(c) Suppose Abe, Bart, Carol, and Denise compete in a spelling bee and the two top finishers will go to the regional finals. In how many possible ways could two of these four students be chosen to go to the regional finals. Check by listing all possible combinations for the top two finishers.

Answer.  (a) The number of lunch menus possible is $(3)(2)(10)(6)(4) = 1440$ which would let you have a different meal each day for almost 4 years.

(b) $P_{4,2} = \frac{4!}{2!} = 4 \cdot 3 = 12$. To list all 12 possibilities, list each possible winner with each possible 2nd place finisher:
AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC

(c) The number of possible combinations is $C_{4,2} = \frac{4!}{2!2!} = \frac{4 \cdot 3}{2} = 6$. In this case, order of finish doesn’t matter, so, e.g. CD and DC are equivalent and so on. In this case the six possibilities are
AB, AC, AD, BC, BD, CD

2. An assignment contains 20 questions.

(a) In how many ways can the grader choose 10 of those 20 questions to grade?

(b) Suppose Winton did 17 of the 20 questions. What is the probability that Winton did all of the 10 questions the grader will randomly choose to grade?

Answer.  (a) $C_{20,10} = \frac{20!}{10!10!} = 184,756$.

(b) The number of groups of 10 problems Winton did is $C_{17,10} = 19,448$. Therefore, the probability that one of Winton’s groups of 10 is the group of 10 that a grader will randomly choose is

$$\frac{19,448}{184,756} = .1053$$
3. (From earlier topics) In a certain community approximately 20% of senior citizens (65 or older) get the flu each year. However, about 30% of people under 65 get the flu each year. Also, approximately 25% of the community is composed of senior citizens.

(a) What is the probability that a person selected at random from the community is a senior citizen who will get the flu this year?

(b) What is the probability that a person selected at random from the community is a person under age 65 who will get the flu this year?

(c) What is the probability that a person selected at random from the community will get the flu this year?

(d) Are the events an individual is a senior citizen and the event an individual will get the flu this year independent? Explain. Are they mutually exclusive?

(e) What is the probability that a person selected at random from the community will get the flu this year or is a senior citizen?

**Answer.** Let $F$ be the event a randomly selected individual will get the flu this year, let $S$ be the event the selected individual is a senior and let $Y$ be the event the selected individual is not a senior.

(a) $P(F \text{ and } S) = P(S) \cdot P(F|S) = (.25)(.20) = .05$.

(b) $P(F \text{ and } Y) = P(Y) \cdot P(F|Y) = (.75)(.30) = .225$.

(c) $P(F) = .05 + .225 = .275$

(d) The events $F$ and $S$ are not independent because $P(F) \neq P(F|S)$. The probability of someone getting the flu this year depends on whether they are a senior or not. The events $F$ and $S$ are not mutually exclusive, because seniors do get the flu, i.e. $P(F \text{ and } S) = .05 \neq 0$.

(e) $P(F \text{ or } S) = P(F) + P(S) - P(F \text{ and } S) = .275 + .25 - .05 = .475$. 