Name.  

**Hints and Answers**

1. Probability distribution for $x$ the number of prisoners out of five on parole who become repeat offenders.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>.237</td>
<td>.396</td>
<td>.264</td>
<td>.088</td>
<td>.015</td>
<td>.001</td>
</tr>
</tbody>
</table>

(a) Find the probability that one or more of the five parolees will be repeat offenders.

(b) Compute $\mu$ the expected number of repeat offenders out of five.

(c) Compute $\sigma$ the standard deviation for the number of repeat offenders out of five.

**Answer.**

(a) $P(x \geq 1) = 1 - P(x = 0) = 1 - .237 = .763$

(b) 

$$
\mu = .396 + 2(.264) + 3(.088) + 4(.015) + 5(.001) \\
= 1.253.
$$

(c) $\sigma$ is computed as

$$
\sqrt{.396 + 2^2(.264) + 3^2(.088) + 4^2(.015) + 5^2(.001)} - 1.253^2
$$

which is $\sqrt{.938881} = .969$.

2. Random variables $x_1$ and $x_2$ represent, respectively, the length of time in minutes for a company to check the computer and the length of time for a company to repair the computer. For these random variables it is know that

$x_1$: $\mu_1 = 28.1$ $\sigma_1 = 8.2$

$x_2$: $\mu_2 = 90.5$ $\sigma_2 = 15.2$

(a) Let $W = x_1 + x_2$. Compute the mean, variance and standard deviation for $W$.

(b) Suppose it costs $1.50 per minute to examine the computer and $2.75 per minute to repair the computer. Then $W = 1.50x_1 + 2.75x_2$ represents the service cost. Compute the mean, variance and standard deviation for $W$.

(c) Suppose $L = 1.5x_1 + 50$. Compute the mean, variance and standard deviation for $L$.

**Answer.**

(a) $\mu_W = \mu_1 + \mu_2 = 28.1 + 90.5 = 118.6$.

$\sigma^2_W = \sigma^2_1 + \sigma^2_2 = 8.2^2 + 15.2^2 = 298.28$

$\sigma_W = \sqrt{298.28} = 17.27$.

(b) Compute $\mu_W = 1.5\mu_1 + 2.75\mu_2 = 291.025$ and

$$
\sigma^2_W = 1.5^2\sigma^2_1 + 2.75^2\sigma^2_2 \\
= (1.5)^2(8.2)^2 + (2.75)^2(15.2)^2 = 1898.53
$$

Thus $\sigma_W = \sqrt{1898.53} = 43.57$.

(c) Compute $\mu_L = 1.5\mu_1 + 50$ and $\sigma^2_L = 1.5^2\sigma^2_1$ and so $\mu_L = 92.15$ and $\sigma^2_L = 151.29$ and $\sigma_L = 12.30$. 
3. Suppose a surgeon has a 95% success rate on a certain type of surgery. Suppose the surgeon will perform 20 of these surgeries tomorrow. Use the binomial distribution to find

(a) The probability that exactly 18 of the surgeries will be successful.

**Answer.** \( P(18) = C_{20,18}(.95)^{18}(.05)^{2} = .1887 \)

(b) The probability that exactly 19 of the surgeries will be successful.

**Answer.** \( P(19) = C_{20,19}(.95)^{19}(.05)^{1} = .3774 \)

(c) The probability that all 20 of the surgeries will be successful.

**Answer.** \( P(20) = (.95)^{20} = .3585 \)

(d) The probability that 17 or fewer of the surgeries will be successful.

**Answer.** \( P(r \leq 17) = 1 - [P(20) + P(19) + P(18)] = 1 - (.3585 + .3774 + .1887) = .0755 \)

(e) Find the mean and standard deviation for the number of successful surgeries.

**Answer.** \( \mu = (20)(.95) = 19 \) and \( \sigma = \sqrt{(20)(.95)(.05)} = \sqrt{.95} = .9747 \)

4. Use the table of binomial probabilities to make a probability histogram for the number of correct responses on an 8 question multiple choice test where each question has 4 choices (A,B,C or D) and the test taker is guessing for all answers.

**Hint.** Use the table with \( n = 8, \) and \( p = .25 \) since there is a 1 in 4 chance of guessing the correct answer on each question. The histogram will have bars of width 1 centered on the \( r \) values 0 through 8, the heights of the bars (using the table) are as follows.

- The bar centered at 0 has height .100
- The bar centered at 1 has height .267
- The bar centered at 2 has height .311
- The bar centered at 3 has height .208
- The bar centered at 4 has height .087
- The bar centered at 5 has height .023
- The bar centered at 6 has height .004
- The bar centered at 7 has height .000
- The bar centered at 8 has height .000