Instructions. Please justify your answers by showing all appropriate work. Thank you and good luck!

1. (8 pts) The scores of a standardized test are normally distributed with mean 750 and standard deviation 90.
   (a) What score indicates a percentile rank of 78?

   (b) What proportion of scores are between 570 and 660?

   (c) What proportion of scores are less than 570 or greater than 660?

   (d) What proportion of scores are less than 840?

2. (a) (3 pts) A produce company claims that the mean weight in a large shipment of pumpkins is 25 lbs with a standard deviation of 5 lbs. Assuming this claim is true, what is the probability that a random sample of 75 of these pumpkins would have a mean weight of 26 lbs or more?

   (b) (3 pts) Suppose the standard deviation of a population is $\sigma = 50$. What sample size is needed so that the length of a 95 percent confidence interval is 10? (That is, how large of sample is needed in order to estimate the mean to with $\pm 5$ with 95 percent confidence?)
3. (a) (1 pts) True or False. For a continuous probability distribution function, the total area under the graph of the distribution function is 1.

(b) (1 pt) True or False. If $x$ is a continuous random variable, then $P(a \leq x < b) = P(a < x \leq b)$.

(c) (1 pt) True or False. The standard normal random variable is not a continuous random variable because it can take the values 0, 1, 2 or 3 which are integers.

(d) (1 pt) True or False. If $x$ is a normal random variable with mean $\mu$ and standard deviation $\sigma$, then $y = \frac{x - \mu}{\sigma}$ is a random variable with mean 0.

4. A recent survey of 64 randomly selected gas stations in California found that the mean price for unleaded gasoline is $1.97 per gallon with a sample standard deviation of $.08.

(a) Conduct an hypothesis test at a 1% level of significance to determine whether the mean price for unleaded gasoline in California is more than $1.95 per gallon.

(i) (2 pts) State the null and alternative hypotheses.

(ii) (2 pts) State the rejection region and the conclusion of your test.

(b) (2 pts) Report the $P$-value of your test.

(c) (2 pts) Explain what the $P$-value in (b) means in words that an elementary school student would understand.
5. (a) (2 pts) A sample of size 15 is taken from an approximately normal population with unknown mean and standard deviation, and a two tailed hypothesis test on the mean is conducted at a level of significance of \( \alpha = 0.05 \). State the rejection region.

(b) (3 pts) Explain what type I and type II errors are, and what is the probability of making a type I error in (a)?

6. A recent Gallop poll of 1068 people found that 58\% think that President Bush is doing a good job.

(a) (4 pts) Find a 98\% confidence interval for the proportion of the population that think that President Bush is doing a good job.

(b) (2 pts) Based on your interval in (a), would you be comfortable in saying that less than 60\% of the population think that President Bush is doing a good job? Explain.

(c) (3 pts) If the pollster had wanted to estimate the percentage of the population that think President Bush is doing a good job to within plus or minus 2 percent with 98\% confidence, what size of sample should have been used?