Name: ____________________________

Instructions. Complete each of the following five questions. Please show all appropriate work in your solutions in order to obtain maximum credit. You may use a calculator.

1. (10 pts) Short answers (2pts each).

(a) What is a Type I error in an hypothesis test?

**Answer.** A type I error occurs when the null hypothesis is rejected when it is true.

(b) What is a Type II error in an hypothesis test?

**Answer.** A type II error occurs when one fails to reject the null hypothesis when it is false.

(c) If an hypothesis test reports a $P$-value of .034, should the null hypothesis be rejected at a 5% level of significance? Explain.

**Answer.** Yes, because the $P$-value is smaller than $\alpha$.

(d) If a right-tailed hypothesis test reports a test statistic of $z = 2.11$, what is the $P$-value for the test?

**Answer.** $P$-value is $P(z > 2.11) = 1 - .9826 = .0174$.

(e) What sample size should be used in a population with a standard deviation of $\sigma = 12$ to estimate the population mean to within $\pm 2$ in a 99% confidence interval?

**Answer.**

\[
n = \left( \frac{z_{\alpha/2} \sigma}{E} \right)^2 = \left( \frac{2.58(12)}{2} \right)^2 = 239.63. \text{ Therefore, use } n = 240 \text{ for the sample size.}
\]
2. A recent Gallup poll reported that 47% of adult Americans believe the Bible is inspired by word of God. Moreover, the Gallup organization reported:

“These results are based on telephone interviews with a randomly selected national sample of 1,000 adults, aged 18 and older, conducted May, 2007. For results based on this sample, one can say with 95% confidence that the maximum error attributable to sampling and other random effects is ±3 percentage points. In addition to sampling error, question wording and practical difficulties in conducting surveys can introduce error or bias into the findings of public opinion polls.”

(a) (2 pts) What is the confidence interval that polling organization is suggesting for the proportion of adult Americans who believe the Bible is inspired by word of God?

**Answer.** The interval is $0.44 < p < 0.50$ with a confidence level of 95%.

(b) (3 pts) Using the sample data from the Gallup organization, find a 90% confidence interval for the population proportion of adult Americans who believe the Bible is inspired by word of God.

**Answer.** For this $\hat{p} = 0.47$ and $E = 1.645 \sqrt{\frac{(0.47)(0.53)}{1000}} \approx 0.02596$. Therefore the 90% confidence interval is

$$0.444 < p < 0.496$$

(c) (1 pt) Verify that the conditions necessary for constructing a confidence interval on a proportion are satisfied in this case.

**Answer.** $np \approx n\hat{p} = (1000)(0.47) = 470 > 5 \text{ and } nq \approx n\hat{q} = (1000)(0.53) = 530$ as required.
3. Suppose the weights of adult male grizzly bears are normally distributed with a mean of 850 pounds and a standard deviation of 250 pounds.

(a) (2 pts) What is the probability that a randomly selected adult male grizzly bear will weigh 900 pounds or more?

Answer. \[ P(x > 900) = P\left(z > \frac{900 - 850}{250}\right) = P(z > .2) = 1 - .5793 = .4207 \]

(b) (2 pts) What is the probability that a randomly selected sample of 36 adult male grizzly bears will have a sample mean weight of 900 pounds or more?

Answer. \[ P(\bar{x} > 900) = P\left(z > \frac{900 - 850}{250/\sqrt{36}}\right) = P(z > 1.2) = 1 - .8849 = .1151 \]

(c) (2 pts) What is the probability that a randomly selected sample of 36 adult male grizzly bears will have a sample mean weight between 800 and 900 pounds?

Answer. \[ P(800 < \bar{x} < 900) = P(-1.2 < z < 1.2) = .8849 - .1151 = .7698 \]

4. Suppose a random sample of 49 adult male Siberian tigers has a sample mean weight of 500 pounds with a sample standard deviation of 100 pounds.

(a) (4 pts) Find a 99% confidence interval for the true mean weight of adult male Siberian tigers.

Answer. First \( \sigma \) is not given, so we use the \( t \)-distribution with \( d.f. = 48 \). Using the table (go to \( d.f. = 45 \), we find \( t_c = 2.69 \). Thus \[ E = 2.69 \frac{100}{\sqrt{49}} = 38.42 \]. Hence the 99% confidence interval (to the nearest tenth of a pound) is \[ 461.6 < \mu < 538.4 \]

(b) (1 pts) Describe in words (like a news reporter) what the interval in (a) means.

Answer. We are very confident (99% sure) that the true mean weight of adult male Siberian tigers rounded to the nearest pound is between 462 pounds and 538 pounds.

(c) (1 pt) Based on your answer in (a), would you be convinced that the true mean weight of an adult male Siberian tiger is more than 480 pounds? Explain.

Answer. No, the confidence interval allows for the possibility that the the mean weight is as low as 462 pounds which is less than 480 pounds.
5. Last month a survey of gasoline prices found that the average price for regular gas in Riverside was $3.33 per gallon. However, we have reason to suspect the gas price in Riverside is now lower than this. Conduct an hypothesis test to determine whether the average price for regular gasoline in Riverside is less than 3.33 per gallon, and test at \( \alpha = .05 \)

To do this hypothesis test, prices at 64 randomly selected gas stations were computed to have a sample mean of $3.29. Suppose the population standard deviation is known to be .13.

(a) (2 pts) State the null and alternative hypothesis.

**Answer.** The null hypothesis is \( H_0 : \mu = 3.33 \), and the alternative hypothesis is \( H_1 : \mu < 3.33 \).

(b) (3 pts) Report the \( P \)-value of the test.

**Answer.** The test statistic is \( z = \frac{3.29 - 3.33}{.13/\sqrt{64}} = -2.46 \). Therefore, the \( P \)-value is

\[
P(z < -2.46) = .0052
\]

(c) (2 pts) Should you reject or not reject the null hypothesis? Explain the basis for your decision.

**Answer.** We should reject \( H_0 \), because the \( P \)-value is less than \( \alpha \).

(d) (1 pt) Interpret your conclusion in (c) in ordinary language.

**Answer.** The data very strongly suggests that the mean gas price in Riverside is now less than $3.33 per gallon.