Name: **Hints and Answers**

**Instructions.** Complete each of the following five questions. Please show all appropriate work in your solutions in order to obtain maximum credit. You may use a calculator.

1. (16 pts) Short answers (2pts each).
   
   (a) What is a Type I error in an hypothesis test?
   
   **Answer.** A Type I error occurs when the null hypothesis is rejected when it is true.

   (b) Explain what the level of significance $\alpha$ is for an hypothesis test.

   **Answer.** It is the probability at which the tester is willing to risk making a Type I error.

   (c) In terms of $P$-values and $\alpha$, when should the null hypothesis be rejected?

   **Answer.** When the $P$-value $\leq \alpha$.

   (d) If a two tailed test reports a test statistic of $z = 2.23$, what is the $P$-value for the test?

   **Answer.** $P(z < -2.23) + P(z > 2.23) = 2(0.0129) = 0.0258$

   (e) What value of $z_c$ should be used for a 94% confidence interval?

   **Answer.** Find $z$-value so that $50\% + 94/2\% = 97\%$ of the normal curve is to the left of $z$. Thus $z_{0.94} = 1.88$

   (f) What sample size should be used in a population with a standard deviation of $\sigma = 7.2$ to estimate the population mean to within $\pm 1$ in a 95% confidence interval?

   **Answer.** $n = \left(\frac{z_c \sigma}{E}\right)^2 = \left(\frac{1.96 \cdot 7.2}{1}\right)^2 = 199.14$. Thus use $n = 200$. 
1. (g) What is the standard deviation for the sampling distribution of \( \bar{x} \) based on samples of size \( n = 64 \) for a population with a standard deviation of 18?

**Answer.** \( \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{64}} = \frac{18}{8} = \frac{9}{4} = 2.25 \)

(h) What sample size should the Gallup organization use if it wishes to estimate the percentage of Americans who support troop escalation in Iraq to an accuracy of plus or minus 3 percent 19 times out of 20?

**Answer.** \( n = \frac{1}{4} \left( \frac{z_c}{E} \right)^2 = \frac{1}{4} \left( \frac{1.96}{.03} \right)^2 = 1067.1 \). Thus use \( n = 1068 \).

2. A recent poll reported that 23% of adult Americans surveyed approve of the way the United States has handled the war in Iraq. Moreover, the polling organization reported their methods were as follows.

“These results are based on telephone interviews with a randomly selected national sample of 1,006 adults, aged 18 and older, conducted Feb. 24-26, 2007. For results based on this sample, one can say with 95% confidence that the maximum error attributable to sampling and other random effects is ±3 percentage points. In addition to sampling error, question wording and practical difficulties in conducting surveys can introduce error or bias into the findings of public opinion polls.”

(a) (2 pts) What is the confidence interval that polling organization is suggesting for the proportion of adult Americans favoring the way the U.S. has handled war in Iraq?

**Interval:** \(.20 < p < .26 \)  \hspace{1cm} **Level of Confidence:** \( c = .95 \)

(b) (3 pts) Find a 99% confidence interval for the proportion of adult Americans that approve of the way the United States has handled the war in Iraq.

**Ans:** \(.1958 < p < .2642 \)

First, \( z_c = 2.58 \) and \( E = 2.58 \sqrt{\frac{(p)(1-p)}{1006}} \approx .0342 \). The endpoints of the interval are then \(.23 \pm .0342 \). Therefore, \(.1958 < p < .2642 \) with 99% confidence

(c) (1 pt) Explain (or show) why the conditions necessary for constructing a confidence interval on a proportion are satisfied in this case.

**Answer.** Because both \( np \approx (1006)(.23) > 5 \) and \( nq \approx (1006)(.77) > 5 \).
3. Suppose the mean life span of English Springer Spaniel dogs is normally distributed with a mean of 13 years and a standard deviation of 1.5 years.

(a) (2 pts) What is the probability that a randomly selected English Springer Spaniel will live to be 14 years or older?

Ans: \( \frac{1}{2514} \)

Answer. \( P(x > 14) = P \left( z > \frac{14 - 13}{1.5} \right) = P(z > 0.67) = 1 - 0.7486 = 0.2514 \)

(b) (2 pts) What is the probability that a randomly selected sample of 25 English Springer Spaniels will have a mean life span of 14 years or more?

Ans: \( \frac{1}{0004} \)

Answer. \( P(\bar{x} > 14) = P \left( z > \frac{14 - 13}{1.5/\sqrt{25}} \right) = P(z > 3.33) = 1 - 0.9996 = 0.0004 \)

(c) (2 pts) What is the probability that a randomly selected sample of 25 English Springer Spaniels will have a mean life span between 12.5 and 14 years?

Ans: \( \frac{9521}{} \)

Answer. \( P(11.5 < \bar{x} < 14) = P \left( \frac{12.5 - 13}{1.5/\sqrt{25}} < z < 3.33 \right) = P(-1.67 < z < 3.33) = 0.9996 - 0.0475 = 0.9521 \)

4. A government official wishes to determine if there has been “grade inflation” for graduating seniors in her state’s high schools over the last 10 years. So she took random sample of 900 graduating seniors GPA’s in 1996 and found the sample to have a mean GPA of 3.13 and a standard deviation of .56, and she found a sample of 1225 graduating seniors in 2006 had a mean GPA of 3.29 with a standard deviation of .53.

(a) (3 pts) Help this official by constructing a 99% confidence interval for the difference of the population means, use the 1996 GPA’s as population 1. Assume the standard deviations given are also the population standard deviations.

Ans: \( -0.222 < \mu_1 - \mu_2 < -0.098 \)

Answer. \( E = 2.58 \sqrt{\frac{(0.56)^2}{900} + \frac{(0.53)^2}{1225}} \approx 0.062 \). The endpoints of the interval are \( \bar{x}_1 - \bar{x}_2 \pm E \) and so

\( -0.222 < \mu_1 - \mu_2 < -0.098 \) with 99% confidence.

(b) (2 pts) Describe in words (like a news reporter) what the interval in (a) means.

Answer. We are 99% confident that the average GPA of high school graduates in the state in 1996 were .222 to .098 lower than the average GPA of high school graduates in the state in 2006.

(c) (1 pt) Based on your answer in (a), would you be convinced that there has been grade inflation in that state? Explain.

Answer. Yes, we are 99% certain that the GPA’s are at least .098 higher in 2006 than they were in 1996.
5. The recent survey of gasoline prices found that the average price for regular gas in Riverside was $2.838 per gallon. However, we have reason to suspect the gas price in Riverside is higher than this. Conduct an hypothesis test to determine whether the average price for regular gasoline in Riverside is more than 2.838 per gallon, and test at $\alpha = .05$

To do this hypothesis test, prices at 34 randomly selected gas stations were computed to have a sample mean of $2.891$ with a sample standard deviation of .19.

(a) (2 pts) State the null and alternative hypothesis.

**Null Hypothesis:** $\mu = 2.838$

**Alternative Hypothesis:** $\mu > 2.838$

(b) (3 pts) Report the P-value of the test

**P-value:** $0.050 < \text{P-value} < 0.075$

**Answer.** Because the population standard deviation is not known, we will use the $t$-distribution with $d.f. = 33$. First,

$$t = \frac{2.891 - 2.838}{0.19} \approx 1.627.$$  

Thus, from the table $0.050 < \text{P-value} < 0.075$.

(c) (2 pts) Should you reject or not reject the null hypothesis? Explain basis for your decision.

**Answer.** You should not reject the null hypothesis because the P-value is larger than the level of significance of .05.

(d) (1 pt) Interpret your conclusion in (c) in ordinary language.

**Answer.** The data is not strong enough to suggest that true average price for gasoline in Riverside is more than $2.838$ per gallon (at the 5% level of significance). In other words, we are not 95% sure that the average gas price in Riverside is more than $2.838$ per gallon.