Math 121, Chapter 2 Practice Problems
Hints and Answers

1. (a) Find the distance between the points \((-3, 2)\) and \((9, -3)\).

**Answer.** \[d = \sqrt{(9 - (-3))^2 + (-3 - 2)^2} = \sqrt{144 + 25} = 13.\]

(b) find the midpoint of the line segment with endpoints \((3, 5)\) and \((-5, 2)\).

**Answer.** \[\left(\frac{3 + (-5)}{2}, \frac{5 + 2}{2}\right) = \left(-1, \frac{7}{2}\right).\]

2. (a) Determine the center and radius of the circle whose equation is \(x^2 + y^2 + 10x + 4y + 20 = 0\).

**Answer.** Complete the squares:

\[
x^2 + 10x + 25 + y^2 + 4y + 4 = -20 + 25 + 4 \\
(x + 5)^2 + (y + 2)^2 = 9
\]

Therefore, the center is \((-5, -2)\) and the radius is \(r = 3\).

(b) Write the equation of a circle whose center is \((-5, 1)\) and passes through the point \((3, 1)\).

**Answer.** The radius is the distance from \((-5, 1)\) to \((3, 1)\) which is 8. Therefore, the equation is 

\[(x + 5)^2 + (y - 1)^2 = 64.\]

(c) Find the equation of a circle that has diametral endpoints of \((0, 0)\) and \((6, 8)\). (Hint: the center is the midpoint of the diametral endpoints).

**Answer.** The center is at \((3, 4)\), the radius is \(r = \sqrt{(3 - 0)^2 + (4 - 0)^2} = 5\). Thus the equation is \((x - 3)^2 + (y - 4)^2 = 25.\)

3. Let \(f(x) = 2x^2 + 7\) and \(g(x) = |x - 1|\), find

(a) \((f \circ g)(-5)\)  
(b) \((g \circ f)(x)\)  
(c) \((fg)(0)\)  
(d) \((f + g)(0)\)

**Answer.**

(a) \(f(g(-5)) = f(6) = 2(6^2) + 7 = 72 + 7 = 79.\)

(b) \(g(f(x)) = |f(x) - 1| = |2x^2 + 7 - 1| = 2x^2 + 6\) (we can drop the absolute values because \(2x^2 + 6 \geq 0\) for all values of \(x\).)

(c) and (d): \(f(0) = 7\) and \(g(0) = 1\), therefore \((fg)(0) = 7(1) = 7\) and \((f + g)(0) = 7 + 1 = 8.\)
4. Let \( f(x) = 4x^2 - 3x \), find the difference quotient

\[
\frac{f(x + h) - f(x)}{h}
\]

Answer.

\[
\frac{f(x + h) - f(x)}{h} = \frac{4(x + h)^2 - 3(x + h) - (4x^2 - 3x)}{h}
\]

\[
= \frac{4x^2 + 8xh + 4h^2 - 3x - 3h - 4x^2 + 3x}{h}
\]

\[
= \frac{8xh + 4h^2 - 3h}{h}
\]

\[
= \frac{h(8x + 4h - 3)}{h}
\]

\[
= 8x + 4h - 3.
\]

5. Sketch the graph of \( f(x) = |x + 3| - 2 \) and find intervals where \( f \) is (a) increasing; (b) decreasing. Is \( f \) one-to-one?

Answer. Notice that the graph of \( f(x) \) is the graph of \( y = |x| \) shifted left 3 units, and down 2 units. Thus, \( f \) is decreasing on \((-\infty, -3]\) and increasing on \([-3, \infty)\). The sketch is left to the reader. Notice that \( f \) is not one-to-one because it fails the horizontal line test.

6. Determine the domains of the following functions.

(a) \( f(x) = \frac{x + 3}{(x + 2)\sqrt{16 - x^2}} \)

(b) \( g(x) = \sqrt{x - 4} \)

(c) \( h(x) = \sqrt{4 - x} \)

(d) \( k(x) = \frac{3(x - 1)}{(x + 2)(x - 11)} \)

Answer. (a) We need \( 16 - x^2 > 0 \) and \( x + 2 \neq 0 \). Therefore, \(-4 < x < 4 \) and \( x \neq 2 \) which in interval form is \((-4, -2) \cup (-2, 4)\).

(b) \( x \geq 4 \); (c) \( x \leq 4 \) (d) \( x \neq -2, x \neq 11 \).
7. (a) Find the slope-intercept form of the line through the points \((-1, 3)\) and \((4, -7)\).

**Answer.** The slope is \(m = \frac{-7-3}{4-(-1)} = -2\). Using the point-slope form of a line, we get \(y - 3 = -2(x + 1)\) and so \(y = -2x + 1\) is the slope-intercept equation of the line. One should plug both points in to make sure they work.

(b) Find the slope-intercept form of the line that passes through the point \((-3, -7)\) and is perpendicular to the line \(2x + 5y = 10\).

**Answer.** First, the line \(2x + 5y = 10\) has slope \(m_1 = -\frac{2}{5}\). So the perpendicular line should have slope \(m_2 = -1/m_1 = \frac{5}{2}\). Thus the desired line has point-slope form \(y + 7 = \frac{5}{2}(x - (-3))\) which leads to the slope-intercept equation \(y = \frac{5}{2}x - \frac{41}{7}\).

(c) Find the slope-intercept form of the line that passes through the point \((-3, -7)\) and is parallel to the line \(2x + 5y = 10\).

**Answer.** From (b), we know the slope of the line is \(m = -\frac{2}{5}\), and so the parallel line has equation \(y - (-7) = -\frac{2}{5}(x - (-3))\) and so \(y = -\frac{2}{5}x - \frac{41}{7}\).

8. (a) Write the quadratic function \(f(x) = -3x^2 + 4x - 5\) in standard form by completing the square. Using that information, sketch the graph of \(f(x)\).

**Answer.** The completion of the square is \(f(x) = -3x^2 + 4x - 5\)
\[\begin{align*}
&= -3\left(x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2\right) - 5 + 3\left(\frac{2}{3}\right)^2 \\
&= -3\left(x - \frac{2}{3}\right)^2 - \frac{11}{3}.
\end{align*}\]
Thus the graph of \(f(x)\) is the graph of \(y = -3x^2\) shifted \(\frac{2}{3}\) units to the right, and \(\frac{11}{3}\) units down. See text or graphing utility for graph.

(b) Find the vertex of the quadratic function \(f(x) = 3x^2 - 6x + 11\), and find the range of \(f(x)\).

**Answer.** The vertex has \(x\)-coordinate \(-\frac{b}{2a}\), which is \(x = 1\), and so the \(y\)-coordinate of the vertex is \(f(1) = 3 - 6 + 11 = 8\). Thus the vertex is at \((1, 8)\), and the range of \(f\) is \([8, \infty)\) (the parabola opens upward so the vertex is at the minimum).

(c) Find the maximum of the quadratic function \(f(x) = -3x^2 + 3x + 7\) and then find its range.

**Answer.** The maximum occurs at the vertex which is at \(x = 1/2\). Thus the maximum is \(f(1/2) = -3/4 + 3/2 + 7 = 31/4\). The range is \((-\infty, 31/4]\).

(d) Find the range of the quadratic function \(f(x) = x^2 - 10x + 3\). Does this function have a maximum or a minimum? If so, find it.

**Answer.** This is a parabola opening up, so it has a minimum but no maximum. The minimum is at the vertex which is at \((5, f(5))\), i.e. \((5, -22)\). Thus the minimum is \(f(5) = -22\) and the range is \([-22, \infty)\).
9. An air freight company has determined that its cost of delivering \( x \) parcels per flight is 
\[
C(x) = 875 + 0.75x
\]
and it charges $12.00 per parcel to send each parcel. Find:

(a) the revenue function;

**Answer.** \( R(x) = 12x \).

(b) the profit function;

**Answer.** \( P(x) = 12x - (0.75x + 875) = 11.25x - 875 \).

(c) the minimum number of parcels the company must ship on a flight to break even.

**Answer.** Solve \( P(x) \geq 0 \), i.e., \( 11.25x - 875 \geq 0 \) implies \( x \geq \frac{875}{11.25} \approx 77.8 \). Thus the company must ship at least 78 parcels to break even or make a profit.

10. The height in feet of a projectile with an initial velocity of 64 feet per second and an initial height of 80 feet is a function of time \( t \) in seconds, given by
\[
h(t) = -16t^2 + 64t + 80.
\]

(a) Find the maximum height of the projectile.

**Answer.** Using the answer in (b), we compute \( h(2) = -64 + 128 + 80 = 144 \) feet.

(b) Find the time \( t \) when the projectile reaches its maximum height.

**Answer.** The maximum height occurs at \( t = \frac{-64}{2(-16)} = 2 \); that is 2 seconds into its flight.

(c) Find the time \( t \) when the projectile hits the ground (has a height of 0 feet).

**Answer.** \( h(t) = 0 \) implies \(-16(t^2 - 4t - 5) = 0 \), and so \((t - 5)(t + 1) = 0 \). Therefore, the projectile lands after 5 seconds.

(d) The difference quotient \( \frac{h(1.01) - h(0.99)}{1.01 - 0.99} \) gives the average velocity of the projectile for \(.99 \leq t \leq 1.01 \). Compute this difference quotient. Do you think it would provide a good estimate of the instantaneous velocity of the projectile when \( t = 1 \)?

**Answer.** Using a calculator, \( \frac{h(1.01) - h(0.99)}{1.01 - 0.99} = 32 \) feet/second. It should be a good estimate, because the average velocity of a very small time interval should be close to the instantaneous velocity at the time in the midpoint of the interval.
11. (a) Determine whether the function \( f(x) = x^4 - 3x^2 + 10 \) is even, odd, or neither. What about the function \( h(x) = -3x^5 - 7x + 1 \)?

**Answer.** \( f \) is even because \( f(-x) = (-x)^4 - 3(-x)^2 + 10 = x^4 - 3x^2 + 10 = f(x) \). However, \( h \) is neither even nor odd because \( h(-x) \neq h(x) \) and \( h(-x) \neq -h(x) \).

(b) Determine whether the graph of \( y = x^3 - 4x \) is symmetric about the (i) \( x \)-axis, (ii) \( y \)-axis, (iii) origin.

**Answer.** Symmetric to the origin since the equation is unchanged if \( x \) is replaced with \(-x\) and \( y \) is replaced with \(-y\). Please check this.

(c) Determine whether the function \( g(x) = x^5 - x^3 \) is even, odd or neither.

**Answer.** Odd, because \( g(-x) = -x^5 + x^3 = -(x^5 - x^3) = -g(x) \).

(d) In terms of shifts or translations, how does the graph of \( y = f(x + 5) - 10 \) compare to the graph of \( y = f(x) \)?

**Answer.** The graph of \( y = f(x + 5) - 10 \) is the graph of \( y = f(x) \) shifted 5 units to the left, and 10 units down.

(e) In terms of shifts or translations, how does the graph of \( y = f(x + 5) - 10 \) compare to the graph of \( y = f(x - 3) + 2 \)?

**Answer.** Shift the graph of \( y = f(x - 3) + 2 \) to the left 8 units and down 12 units to get the graph of \( y = f(x + 5) - 10 \).

12. Find two numbers whose difference is 10 and the sum of whose squares is a minimum.

**Answer.** Let the two numbers be \( x \) and \( y \). Then \( y - x = 10 \) and so \( y = x + 10 \). Now we minimize \( x^2 + y^2 = x^2 + (x + 10)^2 \). In other word, we minimize the quadratic function \( f(x) = 2x^2 + 20x + 100 \). The minimum occurs when \( x = -\frac{b}{2a} = -\frac{20}{4} = -5 \). Thus \( x = -5 \) and \( y = -5 + 10 = 5 \). So the two numbers are \(-5\) and \(5\), and the sum of their squares is 50.

13. Let \( f(x) = \sqrt{5 - x} \) and \( g(x) = \sqrt{x + 7} \). Find the domain of (i) \( f + g \), (ii) \( f - g \), (iii) \( fg \), (iv) \( f/g \).

**Answer.** Remember, the domains of \( f + g \), \( f - g \) and \( fg \) are all the same, and they are the intersection of the domains of \( f \) and \( g \). The domain of \( f \) is \((-\infty, 5]\) and the domain of \( g \) is \([-7, \infty)\). The intersection of these domains is \([-7, 5]\) which is the answer for (i), (ii), and (iii).

For (iv), the domain is all \( x \in [-7, 5] \) such that \( g(x) \neq 0 \). Now \( g(x) = 0 \) if \( x = -7 \). Therefore, the domain of \( f/g \) is \((-7, 5]\).
14. A farmer has $1000 to spend to fence a rectangular corral. Because extra reinforcement is needed on one side, the corral costs $6 per foot along that side. It costs $2 per foot to fence the remaining sides. What dimensions of the corral will maximize the area of the corral?

**Answer.** Let the dimensions be \( x \) and \( y \), with the \( y \) being the length of the expensive side. Then \( 2x + 2x + 2y + 6y = 1000 \). Therefore, \( 4x + 8y = 1000 \) and so \( x = 250 - 2y \). Now we maximize the area \( xy = y(250 - 2y) \). So we maximize the quadratic function \( f(y) = -2y^2 + 250y \). This maximum occurs when \( y = \frac{-250}{-4} = 62.5 \) feet and so \( x = 250 - 125 = 125 \) feet. Thus the dimensions are 62.5 feet by 125 feet, where the expensive side is 62.5 feet long.

15. A Hollywood charter bus company that provides tours through the movie star neighborhoods in Beverly Hills has determined that the cost of providing \( x \) people a tour is

\[
C(x) = 180 + 2.50x
\]

A full tour consists of 60 people. The ticket price per person is $15 plus $0.25 for each unsold ticket. Determine

(a) The revenue function.

(b) The profit function.

(c) The company’s maximum profit.

(d) The number of ticket sales that yields the maximum profit.

**Answer.**

(a) \( R(x) = x(15 + .25(60 - x)) = -.25x^2 + 30x \).

(b) \( P(x) = R(x) - C(x) = -.25x^2 + 27.5x - 180 \).

(d) The number of tickets is \( -27.5/(2(-.25)) = 55 \).

(c) The maximum profit is \( P(55) = -.25(55^2) + 27.5(55) - 180 = 576.25 \).

16. Answer the following in terms of shifts, reflections, stretching or shrinking.

(a) How does the graph of \( y = f(-x) \) relate to the graph of \( y = f(x) \)?

(b) How does the graph of \( y = -f(x) \) relate to the graph of \( y = f(x) \)?

(c) How does the graph of \( y = -f(x + 2) \) relate to the graph of \( y = f(x) \)?

(d) How does the graph of \( y = f(5x) \) relate to the graph of \( y = f(x) \)?

(e) How does the graph of \( y = f(\frac{1}{12}x) \) relate to the graph \( y = f(x) \)?

(f) How does the graph of \( y = 10f(x) \) relate to the graph of \( y = f(x) \)?

**Answer.**

(a) It is a reflection about \( y \)-axis. (b) It is a reflection about \( x \)-axis. (c) Shift graph of \( f \) two units left and then reflect about \( x \)-axis. (d) Horizontally compressed by factor of 1/5 towards \( y \)-axis. (e) Stretched horizontally by factor of 12 away from the \( y \)-axis. (f) Vertically stretched by factor of 10 away from the \( x \)-axis.
17. (a) The function \( I(x) = 12x \) converts feet to inches and the function \( F(x) = 5280x \)
converts miles to feet. Compute \((I \circ F)(x)\) and explain its meaning.
(b) Let \( f(x) = x^2 + 4x - 1 \) and \( g(x) = x + 2 \). Find \( f \circ g \) and \( g \circ f \).
(c) Let \( f(x) = x^2 + 1 \) and \( g(x) = \sqrt{x - 1} \). Compute \( f \circ g \) and \( g \circ f \). What are their domains?
Are \( f \circ g \) and \( g \circ f \) equal?

Answer. (a) \((I \circ F)(x) = I(F(x)) = I(5280x) = 12(5280x) = 63360x\). Converts miles to inches.
(b) \((f \circ g)(x) = f(g(x)) = (x + 2)^2 + 4(x + 2) - 1 = x^2 + 4x + 4 + 4x + 8 - 1 = x^2 + 8x + 11\). On the other hand, \((g \circ f)(x) = (x^2 + 4x - 1) + 2 = x^2 + 4x + 1\).
(c) \((f \circ g)(x) = (\sqrt{x - 1})^2 + 1 \) (note: domain is \( x \geq 1 \)) and so \((f \circ g)(x) = x - 1 + 1 = x\) for \( x \geq 1 \). On the other hand, \((g \circ f)(x) = \sqrt{x^2 + 1 - 1} = \sqrt{x^2} = |x|\) and the domain is \((-\infty, \infty)\).

18. Julie opened a lemonade stand and found that daily her profit is a linear function of the number of cups of lemonade sold. When she sells 300 cups of lemonade, she makes $40 and when she sells 600 cups of lemonade, she makes $130.

(a) Find the profit function.
(b) How many cups of lemonade does Julie need to sell to break even on a given day?
(c) How many cups of lemonade does Julie need to sell to make $100 in a day?
(d) How much would she make on a day when she sells 1000 cups of lemonade?

Answer. (a) Let \( x \) be the number of cups sold, then \( P(x) = mx + b \) where \( m = \frac{130 - 40}{600 - 300} = \frac{90}{300} = 0.3 \).
Thus \( P(x) = .3x + b \), and so \( 40 = .3(300) + b \) which means \( b = -50 \). Hence \( P(x) = .3x - 50 \).
(b) To break even, she must have \(.3x - 50 = 0\), and so \( x = 50/.3 = 166.67 \). That is, she must sell 167 cups of lemonade.
(c) To make $100, we solve \(.3x - 50 = 100\), and so \( x = 150/.3 = 500 \) cups.
(d) She will make \( P(1000) = .3(1000) - 50 = 250 \) dollars.
19. (a) and (d) are even because of symmetry about the $y$-axis.
(e) and (f) are odd because of symmetry about the origin.
(b) and (c) are neither even nor odd, because they are not symmetric about the $y$-axis or origin.

20. (a) (b) (c) (d)
23. In the graph below, the blue graph is a graph of the function \( f(x) = (x + 2)^2(x - 1) \). Find the equation for the green graph.

**Answer.** The green graph is a result of shifting the blue graph 3 units up and 2 units to the left. Therefore, the green graph has equation \( g(x) = f(x+2)+3 \), or \( g(x) = (x+2+2)^2(x-1+2)+3 \) and so \( g(x) = (x + 4)^2(x + 1) + 3 \)

24.
25. (a) (b) (c) (d)

26. (a) (b) (c) (d)
27. (a) 

28. (a)
29. (a) \( y = |x + 2| + 3 \) because the graph of \( y = |x| \) is shifted 2 units to the left and 3 units up.

(b) \( y = |x + 2| - 3 \) because the graph of \( y = |x| \) is shifted 2 units to the left and 3 units down.

(c) \( y = |x - 2| + 3 \) because the graph of \( y = |x| \) is shifted 2 units to the right and 3 units up.

(d) \( y = |x - 2| - 3 \) because the graph of \( y = |x| \) is shifted 2 units to the right and 3 units down.

(e) \( y = -|x - 2| + 3 \) because the graph of \( y = |x| \) is reflected over the \( x \)-axis and then shifted 2 units to the right and 3 units up.

(f) \( y = -|x - 2| \) because the graph of \( y = |x| \) is reflected over the \( x \)-axis and shifted 2 units to the right.

(g) \( y = -|x + 4| + 3 \) because the graph of \( y = |x| \) is reflected over the \( x \)-axis and shifted 4 units to the left and then 3 units up.

(h) \( y = -|x + 4| - 1 \) because the graph of \( y = |x| \) is reflected over the \( x \)-axis and shifted 4 units to the left and then 1 unit down.