1. Let $f(x) = 3x^2 + 4x - 5$. Find and simplify the difference quotient of $f$.

**Answer:** The difference quotient is

$$
\frac{f(x + h) - f(x)}{h} = \frac{3(x + h)^2 + 4(x + h) - 5 - (3x^2 + 4x - 5)}{h}
= \frac{3(x^2 + 2xh + h^2) + 4x + 4h - 5 - 3x^2 - 4x + 5}{h}
= \frac{3x^2 + 6xh + 3h^2 + 4x + 4h - 5 - 3x^2 - 4x + 5}{h}
= \frac{h(6x + 3h + 4)}{h} = 6x + 3h + 4.
$$

Therefore, the difference quotient is $6x + 3h + 4$.

2. Let $f(x) = x^2 - 11x$ and $g(x) = 2x + 3$. Find (a) $f \circ g$ and (b) $g \circ f$.

**Answer:** (a) $(f \circ g)(x) = f(g(x)) = (g(x))^2 - 11(g(x)) = (2x + 3)^2 - 11(2x + 3) = 4x^2 + 12x + 9 - 22x - 33 = 4x^2 - 10x - 24$.

Therefore, $(f \circ g)(x) = 4x^2 - 10x - 24$.

(b) $(g \circ f)(x) = g(f(x)) = 2f(x) + 3 = 2(x^2 - 11x) + 3 = 2x^2 - 22x + 3$.

Therefore, $(g \circ f)(x) = 2x^2 - 22x + 3$.

3. Find the equation of the line that is parallel to the line $3x + 2y = 7$ and passes through the point $(-2, -4)$ . Write your answer in slope-intercept form.

**Answer:** The slope of the parallel line is $m = -3/2$. Therefore, $y + 4 = -\frac{3}{2}(x + 2)$, which in slope-intercept form is:

$$
y = -\frac{3}{2}x - 7
$$
4. (a) Find the equation (in standard form) of the circle \( x^2 + y^2 + 8x + 2y + 8 = 0. \)

**Answer:** Complete the squares:

\[
 x^2 + 8x + 16 + y^2 + 2y + 1 = -8 + 16 + 1
\]

and so the equation of the circle is

\[
 (x + 4)^2 + (y + 1)^2 = 3^2
\]

(b) What is the radius of the circle?  
**Ans:** 3

(c) What is the center of the circle?  
**Ans:** \((-4, -1)\)

5. Consider the points \((-2, 1)\) and \((-4, 5)\).

(a) Find the midpoint of the two points.  
**Ans:** \((-3, 3)\)

The midpoint is: \(\left(\frac{-2 - 4}{2}, \frac{1 + 5}{2}\right) = (-3, 3)\)

(b) Find the distance between the two points.  
**Ans:** \(2\sqrt{5}\)

\[
d = \sqrt{(-4 - (-2))^2 + (5 - 1)^2} = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}
\]

6. Consider the quadratic function \( f(x) = -2x^2 + 12x - 10. \)

(a) Write the function \( f \) in the standard form from: i.e, in the form \( f(x) = a(x - h)^2 + k. \)

**Answer:** \( h = \frac{-b}{2a} \) and so \( h = -12 / (-4) = 3. \) Then \( k = f(3) = -2(3^2) + 12(3) - 10 = 8. \) Therefore,

\[
 f(x) = -2(x - 3)^2 + 8
\]

(b) Find the range of \( f. \) Write your answer in interval notation.

The parabola opens down, and so the range is \((-\infty, k]\), that is

\[
 (-\infty, 8]\]
7. Describe any symmetries (w.r.t. $x$-axis, $y$-axis or origin) the graphs of the following equations possess. Justify your answers.

(a) $x = y^4 + y^2 + 1$

**Answer:** Replacing $y$ with $-y$ yields: $x = (-y)^4 + (-y)^2 + 1$, that is $x = y^4 + y^2 + 1$ which is the original equation. Thus there is symmetry over the $x$-axis. However, replacing $x$ with $-x$, and replacing $(x,y)$ with $(-x,-y)$ both change the equation, so the graph is not symmetric with respect to the $y$-axis or origin.

(b) $|x| + |y| = 10$

**Answer:** Replacing $y$ with $-y$; or $x$ with $-x$; or $(x,y)$ with $(-x,-y)$ all leave the equation unchanged. Thus the graph is symmetric with respect to the $x$-axis, $y$-axis and origin.

8. Consider the piecewise defined function $f(x) = \begin{cases} 11, & \text{if } x < 3; \\ 3x - 2, & \text{if } 3 \leq x \leq 8; \\ 4x - 1, & \text{if } x > 8. \end{cases}$

(a) Find: $f(3)$

**Ans:** 7

(b) Find: $f(5)$

**Ans:** 13

(c) Find $f(-9)$

**Ans:** 11

(d) Find $f(r + 3)$ if $r > 5$

**Ans:** $4r + 11$

**Answer:** Because $r > 5$, then $r + 3 > 8$ and so the last line of the formula should be used. That is, $f(r + 3) = 4(r + 3) - 1 = 4r + 11$.

9. Find the equation of the line through the points $(4, -2)$ and $(2, 8)$. Write your answer in slope-intercept form.

**Answer:** The slope is $m = \frac{8 - (-2)}{2 - 4} = \frac{10}{-2} = -5$. Thus, plugging in $(4, -2)$ we see that $-2 = -5(4) + b$, and so $b = 18$. Therefore, the equation of the line is

$$y = -5x + 18$$
10. A charter boat company has determined that the cost, in dollars, of providing $x$ people a cruise is $C(x) = 500 + 50x$. The ticket price per person in dollars is $190 - x$.

(a) Find the revenue function $R(x)$ for providing a tour for $x$ people

**Answer:** $R(x) = x(190 - x) = -x^2 + 190x$.

(b) Find the profit function $P(x)$ (note: profit is equal to revenue minus cost).

**Answer:** $P(x) = R(x) - C(x) = -x^2 + 190x - (500 + 50x) = -x^2 + 140x - 500$.

(c) Find the number of tickets sales that yields the maximum profit (you do not need to find the maximum profit).

**Answer:** $x = -\frac{b}{2a} = -\frac{-140}{-2} = 70$.

11. Let $f(x) = \sqrt{2x}$.

(a) Describe in terms of translations how to obtain the graph of $g(x) = \sqrt{2(x + 2)} - 1$ from the graph of $f$.

**Answer:** Shift the graph of $f$ 2 units to the left, and 1 unit down to obtain the graph of $g$.

(b) Find the formula of a function $h$ whose graph is obtained by translating the graph of $f$ vertically up two units, and then reflecting it over the $y$-axis.

**Answer:** $h(x) = f(-x) + 2$, that is $h(x) = \sqrt{-2x} + 2$. 
12. Find the difference quotient \( \frac{f(x + h) - f(x)}{h} \) for \( f(x) = 5x - x^2 \).

**Answer:**

\[
\frac{f(x + h) - f(x)}{h} = \frac{5(x + h) - (x + h)^2 - (5x - x^2)}{h} = \frac{5x + 5h - (x^2 + 2xh + h^2) - 5x + x^2}{h} = \frac{5h - 2xh - h^2}{h} = \frac{h(5 - 2x - h)}{h} = 5 - 2x - h
\]

Thus the difference quotient is \( 5 - 2x - h \)

---

13. Let \( f(x) = x^2 - 5x \) and \( g(x) = \sqrt{x + 2} \).

(a) Find the domain of \( \frac{g}{f} \); Write answer in interval notation.

**Answer:** Observe that \( x \geq -2, x \neq 0, 5 \). Thus the domain is \([-2, 0) \cup (0, 5) \cup (5, \infty)\)

(b) Find \( (fg)(2) \)

**Answer:** \( (fg)(2) = (4 - 10)(2) = -12 \)

(c) Find \( (f \circ g)(2) \)

**Answer:** \( f(\circ g)(2) = f(g(2)) = 2^2 - 5(2) = -6 \)