Instructions. Complete question 1 and turn-in this sheet at the end of class for credit.

1. An airplane flies at an altitude of 5 miles toward a point directly over an observer (see the figure below). The speed of the plane is 600 miles per hour. Find the rates at which the angle of elevation \( \theta \) is changing when the angle is (i) \( \theta = 30^\circ \), and (ii) \( \theta = 60^\circ \). Express answer in radians per minute.

**Answer.** Let \( x \) represent the length of the base of the triangle in the figure. Then \( \cot \theta = \frac{x}{5} \); differentiating with respect to \( t \) yields

\[
-csc^2 \theta \cdot \frac{d\theta}{dt} = \frac{1}{5} \cdot \frac{dx}{dt}
\]

we know \( \frac{dx}{dt} = -10 \) miles per minute because the jet is traveling 600 miles per hour. Thus

\[
\frac{d\theta}{dt} = -\frac{1}{5} \sin^2 \theta \cdot \frac{dx}{dt} = 2 \sin^2 \theta
\]

(i) When \( \theta = 30^\circ \), \( \sin \theta = 1/2 \) and so \( \frac{d\theta}{dt} = 2 \sin^2(30^\circ) = \frac{1}{2} \) radian per minute.

(ii) When \( \theta = 60^\circ \), \( \sin \theta = \sqrt{3}/2 \) and so \( \frac{d\theta}{dt} = 2 \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{2} \) radians per minute.

2. A baseball diamond has the shape of a square with sides 90 feet long (see figure). A player runs from second base to third base at a speed of 25 feet per second is 20 feet from third base. At what rate is the player’s distance from home plate changing?

**Answer.** Let \( s \) represent the distance of the runner to home plate and let \( x \) represent the distance of the runner to 3rd base. Then \( x^2 + 90^2 = s^2 \). We differentiate implicitly to find

\[
2x \cdot \frac{dx}{dt} = 2s \cdot \frac{ds}{dt}
\]

and so

\[
\frac{ds}{dt} = \frac{x}{s} \cdot \frac{dx}{dt}
\]

We know that \( x = 20 \) and so \( s = \sqrt{20^2 + 90^2} = 10\sqrt{2^2 + 9^2} = 10\sqrt{85} \), and we know \( dx/dt = -25 \). Therefore,

\[
\frac{ds}{dt} = \frac{20}{10\sqrt{85}} \cdot (-25) = -\frac{50}{\sqrt{85}} = -\frac{10\sqrt{85}}{17} \approx -5.423
\]

The distance between the runner and home plate is decreasing at approximately 5.4 feet per second.